MARCO BALDI
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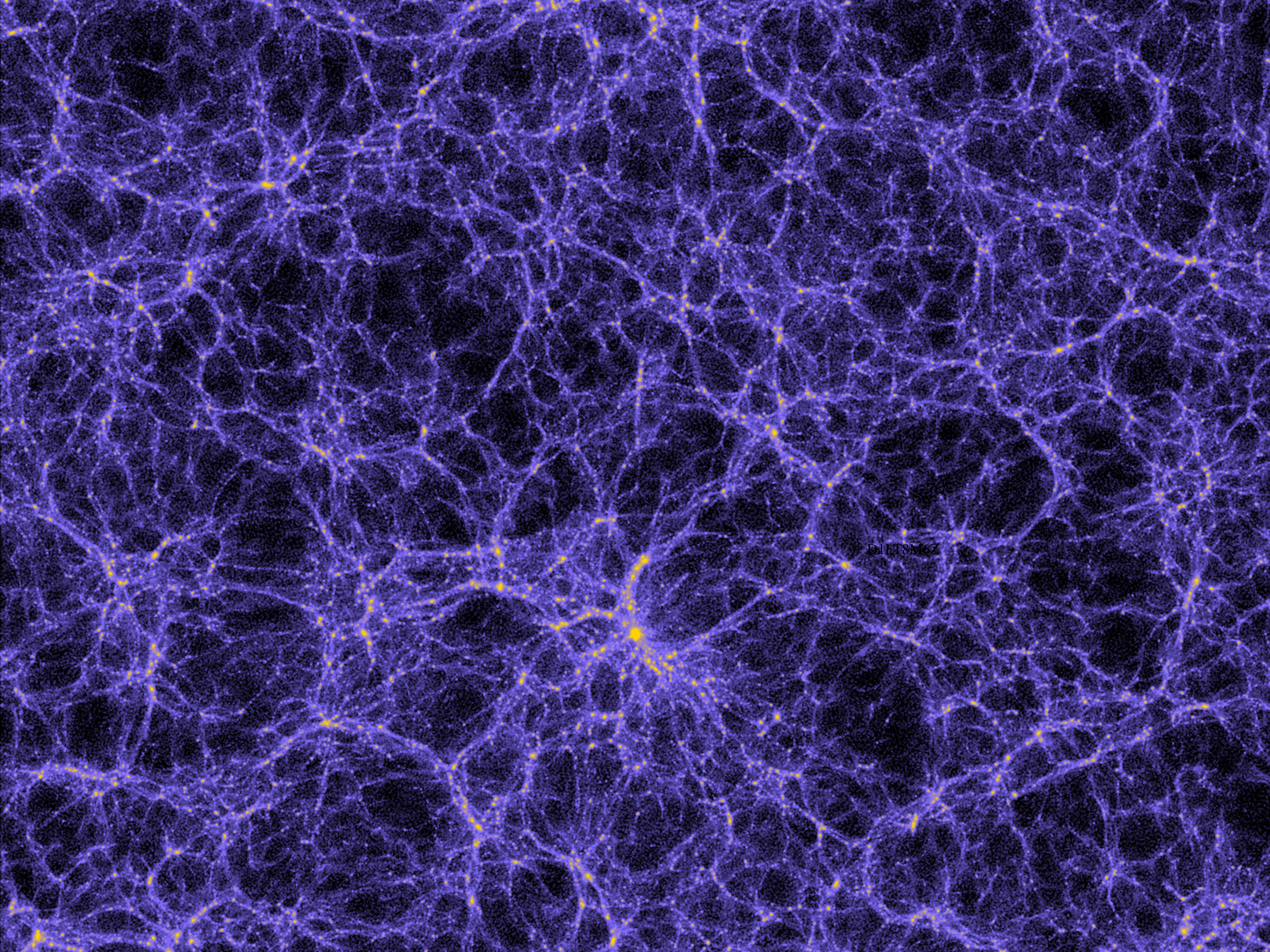
PHYSICS AND ASTRONOMY DEPARTMENT

COSMIC ACCELERATION: FROM THE COSMOLOGICAL CONSTANT TO DARK ENERGY AND MODIFIED GRAVITY THEORIES

Astrophysical probes of fundamental physics
ferrara, 7-11 september 2015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Plan of the course

Three lectures to cover the basics of homogeneous cosmology, perturbations growth, the Dark Energy problem, the cosmological constant and its problems, alternatives to the cosmological constant, and the impact of Dark Energy models on structure formation

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- The cosmological constant

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Lecture 1

- Basics of homogeneous cosmology
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Lecture 2

- Basics of structure formation in the standard model
- Homogeneous Dark Energy models
- Interacting Dark Energy and Modified Gravity

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Lecture 1	Lecture 2	Lecture 3
- Basics of homogeneous cosmology	- Basics of structure formation in the standard model	- Basics of structure formation in DE models
- Observational evidence of cosmic acceleration	- Homogeneous Dark Energy models	- N-body simulations
- The cosmological constant	- Interacting Dark Energy and Modified Gravity	- Non-linear structure formation in DE and modified gravity models

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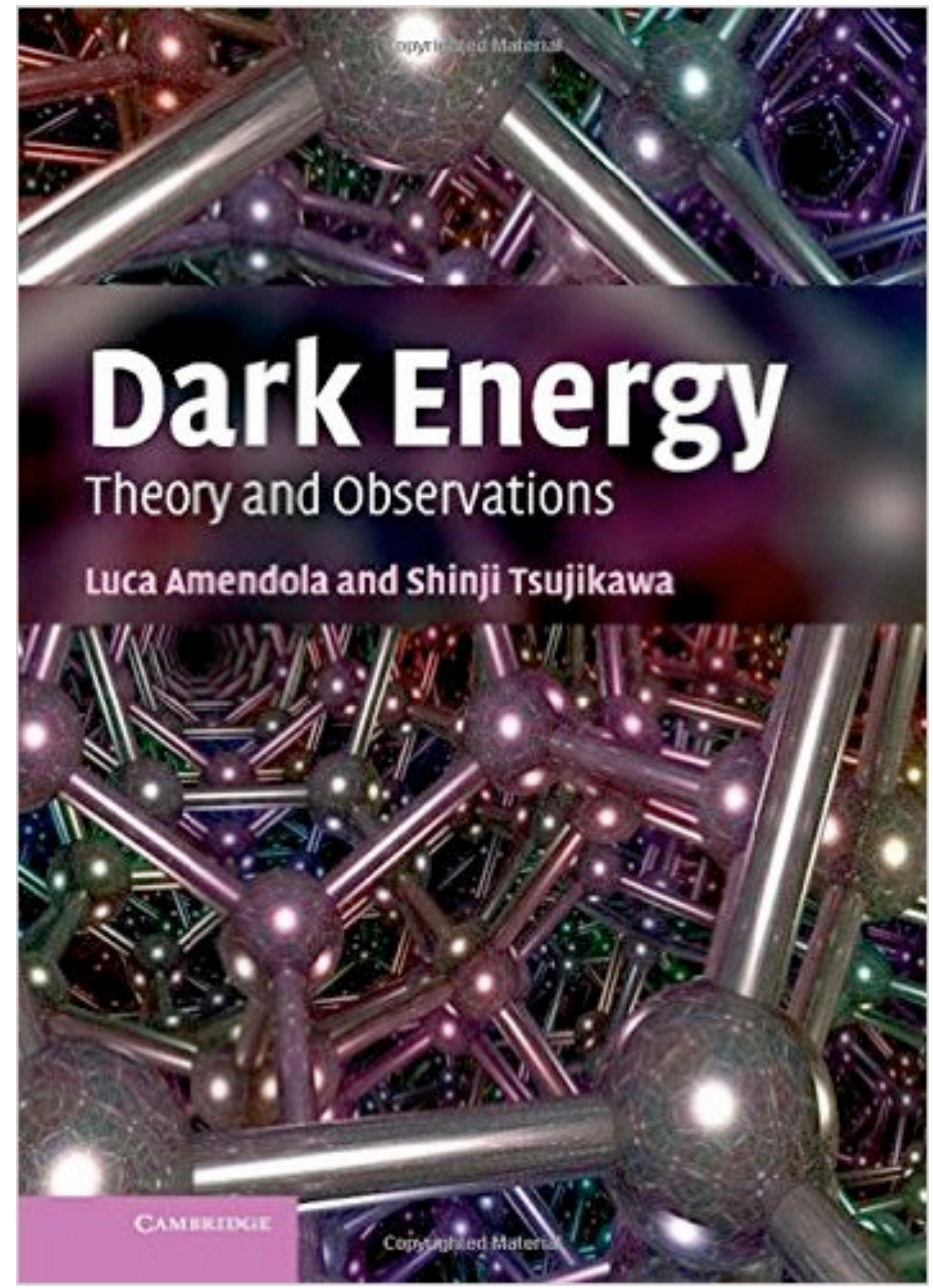
No time to go through the derivations in detail, just showing the path and the main ideas.

Suggested readings

Most of the material presented in these lectures follows the treatment presented in the textbook:

L. Amendola & S. Tsujikawa

Dark Energy
Theory and observations



Suggested readings

Other useful references are:

Textbooks:
- S. Dodelson: Modern Cosmology
- S. Weinberg: Gravitation and Cosmology

Reviews:
- L. Amendola et al. (Euclid Theory WG): arXiv:1206.1225
- M. Baldi: arXiv:1210.6650

Lecture 1

- Basics of homogeneous cosmology
  - *From General Relativity to cosmology*
  - *Friedmann Equations*
  - *Continuity Equations*
  - *Cosmic Inventory: matter species in the Universe*
  - *Acceleration vs. deceleration*

- Observational evidence of Cosmic Acceleration from geometry
  - *Cosmic age vs. stellar age*
  - *Type Ia Supernovae*

- The cosmological constant Λ
  - *Rise and fall of a fascinating concept: the history of Λ*
  - *Problems of the cosmological constant (fine-tuning and coincidence)*

Basics of homogeneous cosmology

From General Relativity to cosmology (I)

Cosmology as a scientific discipline would not exist without the Theory of General Relativity (A. Einstein, 1915... by the way: 1915-2015, happy birthday GR!!!)

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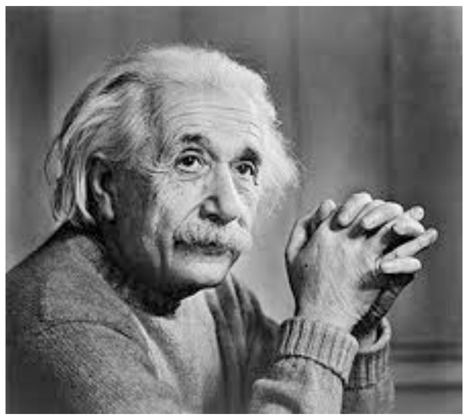



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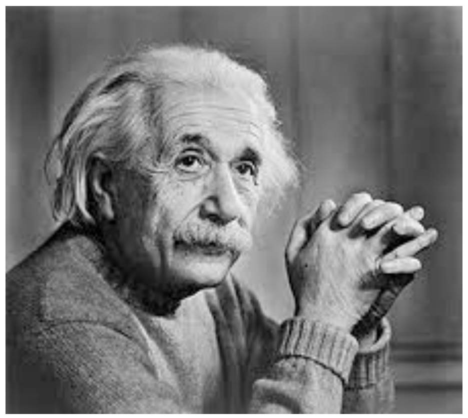    

1915: formulation of the theory of General Relativity (Einstein)

1916: first solution of GR equations for a central mass (Scwarzschild)

1917: first application of GR to a model universe, and introduction of a cosmological constant (Einstein)

1919: confirmation of light deflection from the Sun (Eddington)

1922: first solution of GR equations for an expanding universe with no cosmological constant (Friedmann)

1929: discovery of the cosmic expansion (Hubble)

From General Relativity to cosmology (II)

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where the scalar curvature $R \equiv g^{\mu\nu} R_{\mu\nu}$ and the Ricci tensor ($R_{\mu\nu}$) is defined in terms of the Christoffel symbol $\Gamma^{\mu}_{\nu\lambda}$:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu} \tag{3}$$

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where:

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha} (g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha}) \; ; \quad [*]_{,\xi} \equiv \partial[*]/\partial x^{\xi} \tag{4}$$

From General Relativity to cosmology (III)

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Therefore, the Einstein tensor is fully determined by the metric tensor $g_{\mu\nu}$, which is a unitary tensor ($g^{\mu\alpha}g_{\alpha\nu} = \delta^{\mu}_{\nu}$) defined through the line element in a 4-dimensional space-time:

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \quad (5)$$

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This assumption corresponds to a particular form of the metric tensor called the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{6}$$

for a 3+1 splitting of a time coordinate (the cosmic time $x^0 = t$) and 3 polar spatial coordinates $(x^1, x^2, x^3) = (r, \theta, \phi)$

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$$\Gamma^i_{0j} = \Gamma^i_{j0} = H\delta^i_j\,, \quad \Gamma^1_{11} = \frac{Kr}{1 - Kr^2},\ \Gamma^1_{22} = -r(1 - Kr^2)$$

$$\Gamma^1_{33} = -r(1 - Kr^2 \sin\theta),\ \Gamma^2_{33} = -\sin\theta\cos\theta$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r},\ \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta \qquad (7)$$

where $H \equiv \dot{a}/a$ is the Hubble function and describes the dynamics of the Universe: $H > 0$ = expansion, $H < 0$ = contraction

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- The Cosmological Principle (FLRW) reduces the number of independent Christoffel symbols from 64 to 10.
- The dynamics of the universe is encoded in one function: $a(t)$

Cosmic expansion and Hubble's law

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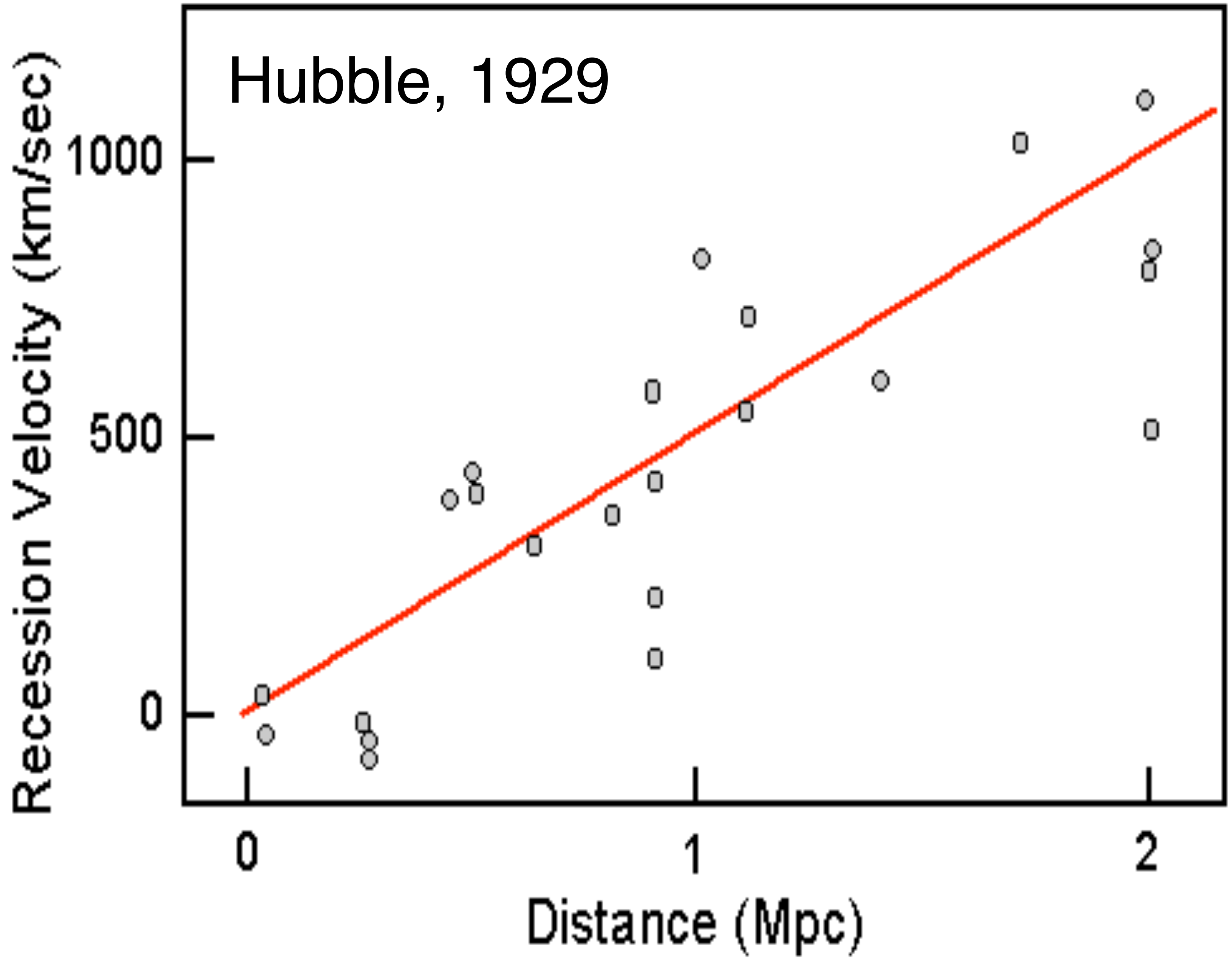
$v \sim H\, d$



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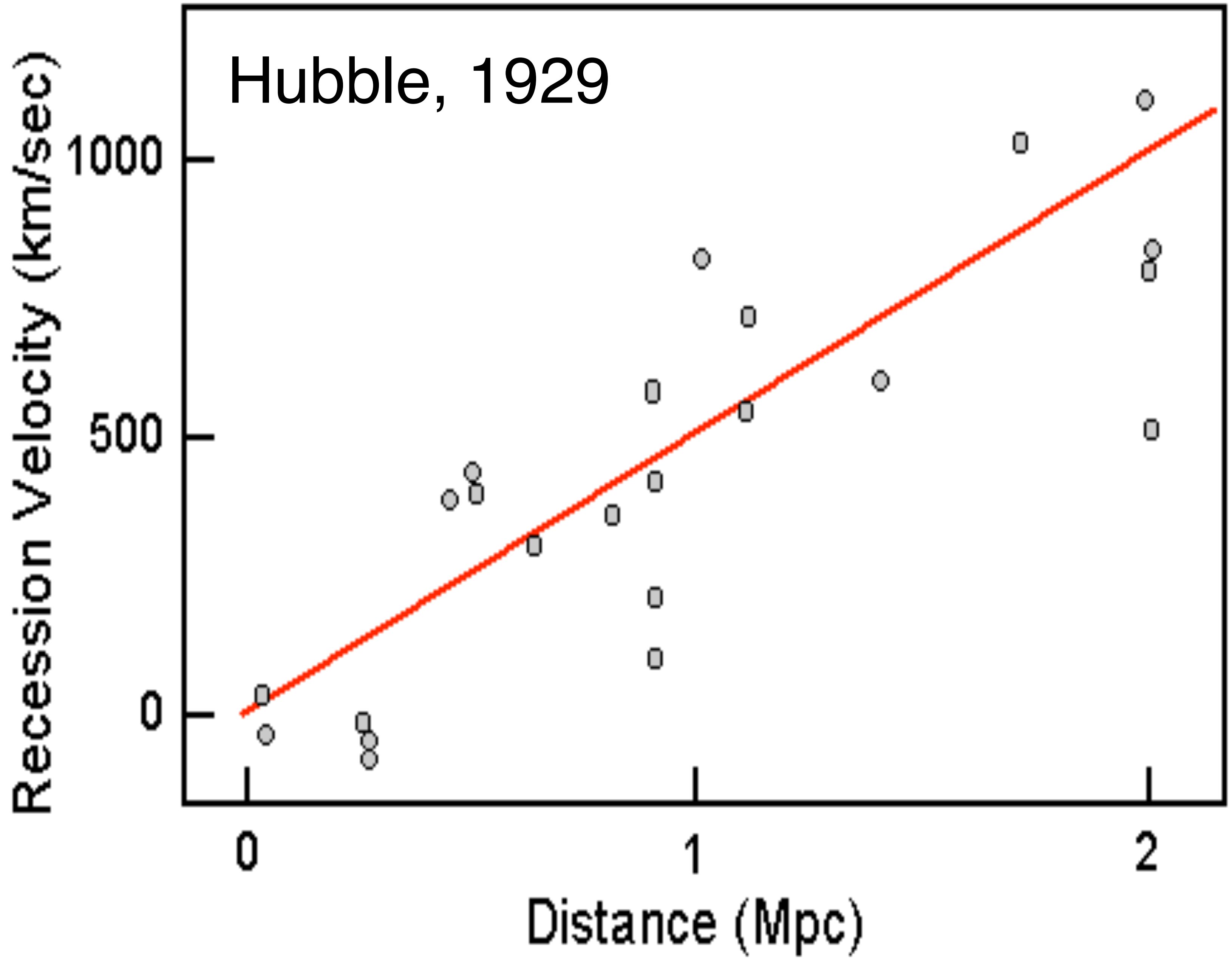
v ~ H d



$$z \equiv \frac{\lambda_0}{\lambda} - 1 = \frac{a_0}{a} - 1 \Rightarrow dt = -dz/[H(1+z)] \quad (8)$$

For $v \ll c$ the Doppler effect gives $\lambda_0 \simeq (1 + v/c)\lambda$
from which $z \simeq v/c$

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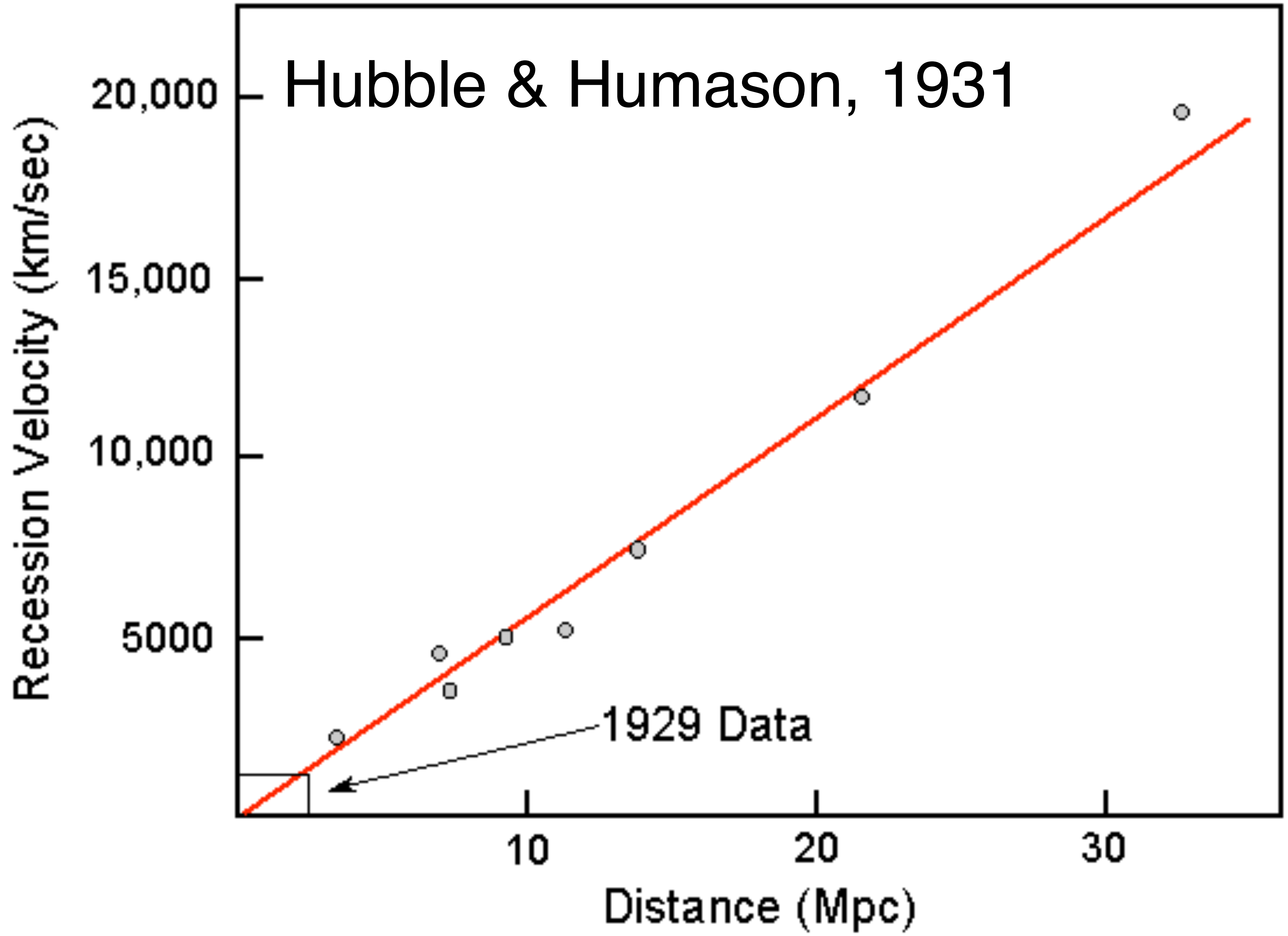

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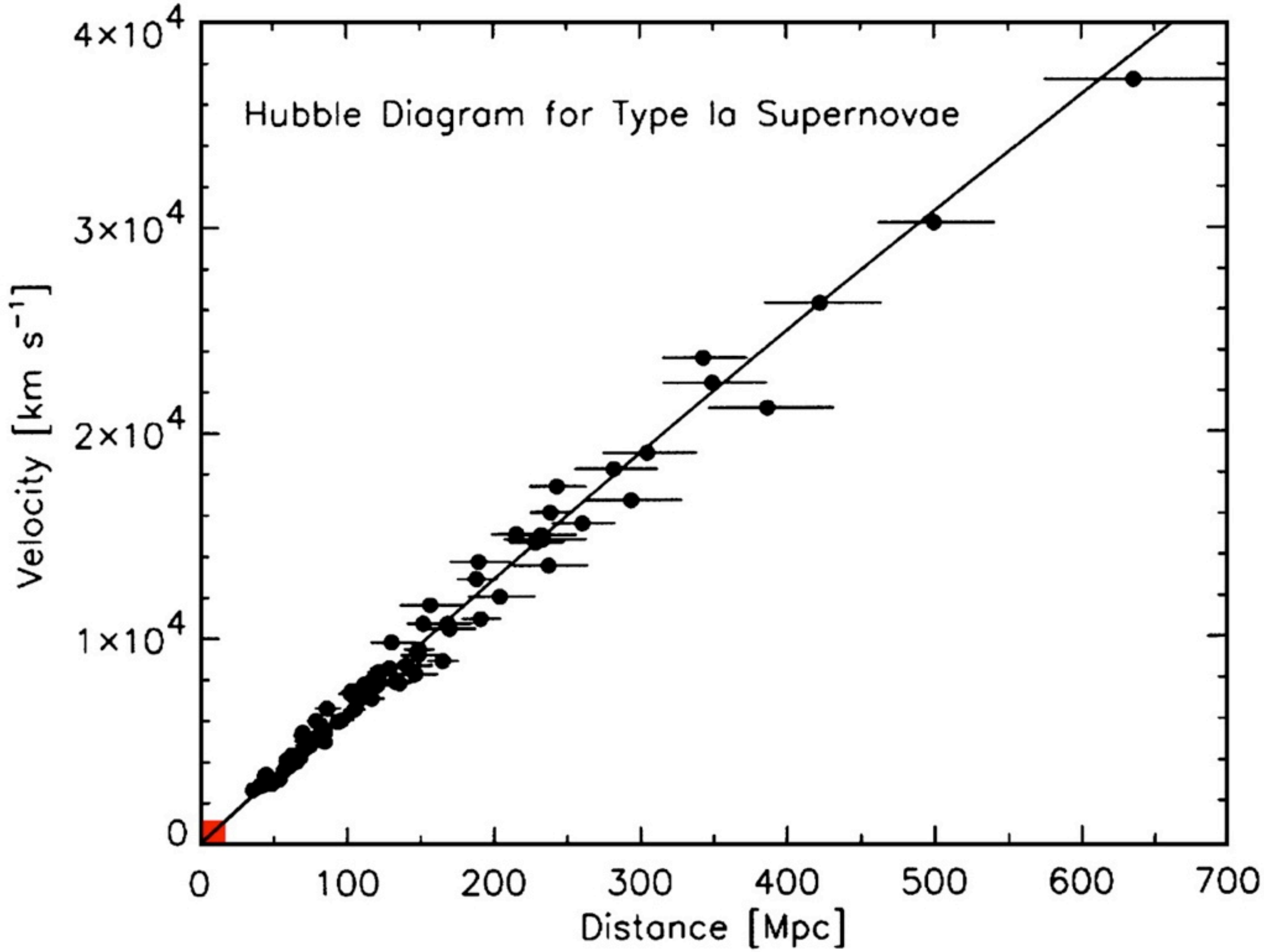

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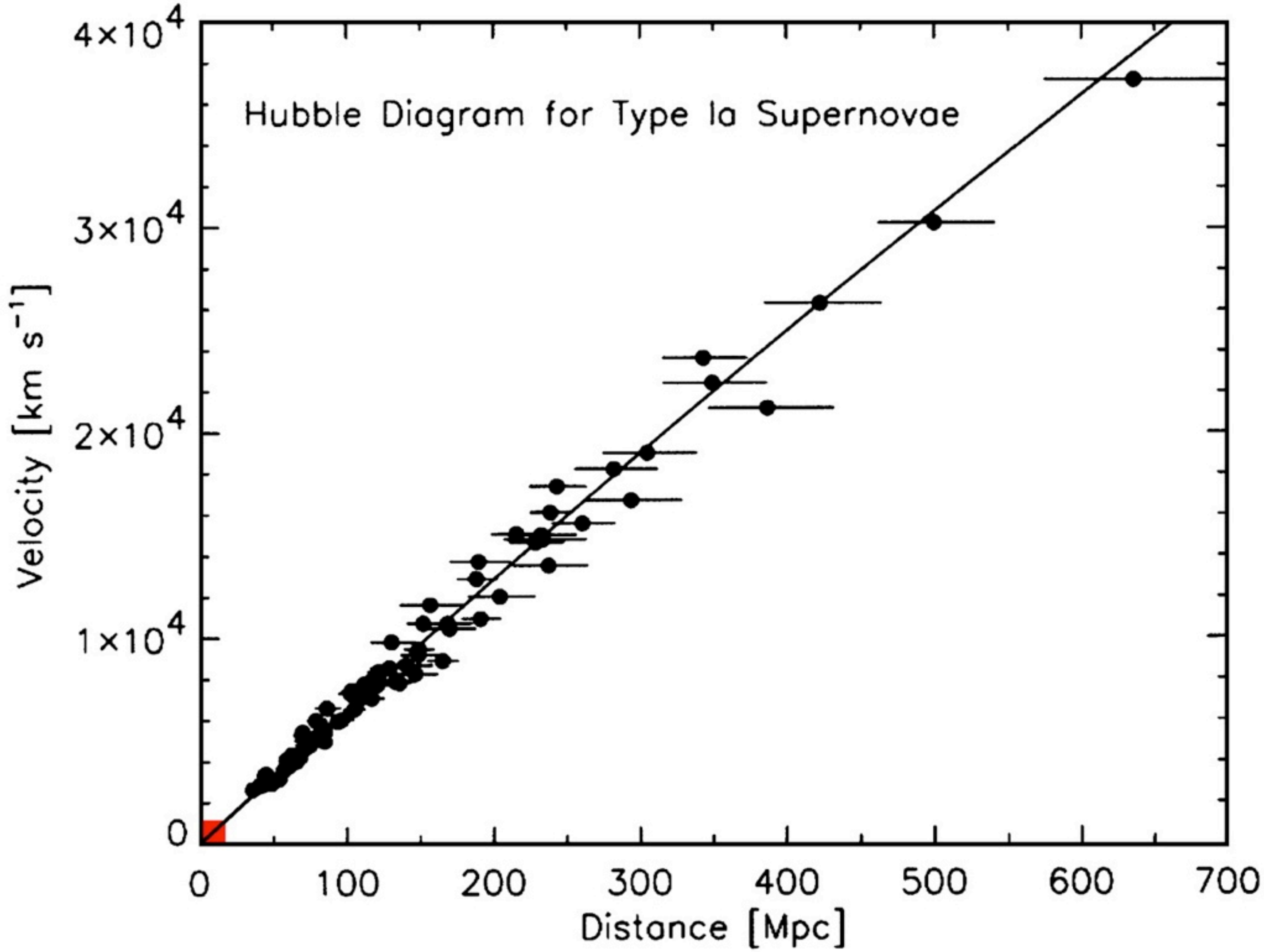

v ~ H d

The distance from an observer at the origin in FLRW is $\vec{r} = a(t)\vec{x}$

From which $\dot{\vec{r}} = H\vec{r} + a\dot{\vec{x}} \equiv \vec{v}_H + \vec{v}_p$ and the radial velocity is:

$$v = Hr + \vec{v}_p \cdot \vec{r}/r \rightarrow \boxed{v \simeq H_0 r} \quad \text{for } z \ll 1 \quad \text{and } v_p \ll Hr \qquad (9)$$

From General Relativity to cosmology (V)

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Substituting the non-vanishing Christoffel symbols in the expression of the Riemann tensor (eq. 3) one gets:

$$R_{00} = -3(H^2 + \dot{H}) \qquad R_{0i} = R_{i0} = 0$$

$$R_{11} = a^2 A \frac{1}{1 - Kr^2} \qquad R_{22} = a^2 A r^2 \qquad R_{33} = a^2 A r^2 \sin^2\theta$$

$$R = 6A \qquad (10)$$

where $A \equiv (3H^2 + \dot{H} + 2K/a^2)$

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$$\rightarrow G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The Friedmann Equations

For a FLRW metric the energy-momentum tensor $T_{\mu\nu}$ can only take the form of a perfect fluid:

$$T^{\mu}_{\nu} = (\rho + p)u^{\mu}u_{\nu} + p\delta^{\mu}_{\nu} \quad (12)$$

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So, from the $(0,0)$ and (i,i) components of the Einstein eqs. (1) one gets the two Friedmann equations:

$$(0,0): \; H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \quad (13)$$

$$(i,i): \; 3H^2 + 2\dot{H} = -8\pi G p - \frac{K}{a^2} \quad \Rightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad (14)$$

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These two equations fully describe the global dynamics of a homogeneous and isotropic Universe.

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By differentiating the first Friedmann eq. (11) and substituting in the second eq. (12) one gets the Continuity Equation:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{15}$$

which describes the evolution of the energy density in an expanding or contracting homogeneous and isotropic Universe.

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from which follows:

$$\nabla_{\mu} T^{\mu}_{\nu} = 0 \tag{17}$$

which gives the same result as (13) for a FLRW metric

Cosmic Inventory: matter species in the Universe

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From the expression of a particle's energy and momentum in Special Relativity (in units of $c = 1$):

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$$f(p) = \frac{1}{\exp[(E - \mu)/T] \pm 1} \tag{19}$$

+ Fermi-Dirac $\quad \mu$ chemical potential
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making use of Heisenberg's Principle $d^3x d^3p \sim (2\pi\hbar)^3$ one gets the expression for the energy and pressure density

$$\rho = g_\star \int \frac{d^3p}{(2\pi\hbar)^3} E(p) f(p) \qquad p = g_\star \int \frac{d^3p}{(2\pi\hbar)^3} \frac{pv}{3} f(p) \tag{20}$$

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In the relativistic ($m \ll T$) and non-relativistic ($m \gg T$) limits this gives:

$$p_r/\rho_r = w_r = 1/3 \qquad p_{nr}/\rho_{nr} = w_{nr} \simeq 0 \tag{21}$$

Deceleration vs. acceleration

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For relativistic ($w_r = 1/3$) or non-relativistic ($w_{nr} \simeq 0$) matter the equation of state w is a constant. This allows to integrate the first Friedmann eq. (11) and the continuity eq. (13) to obtain:

$$\dot{\rho} + 3H(\rho + p) = 0 \quad \Rightarrow \boxed{\rho \propto a^{-3(1+w)}} \tag{22}$$

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \quad \Rightarrow \boxed{a \propto (t - t_i)^{2/[3(1+w)]}} \quad \text{for } K = 0 \tag{23}$$

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these imply that for both matter types the expansion is decelerated:

- relativistic matter $\rightarrow \rho \propto a^{-4};\ a \propto (t - t_i)^{1/2}$
- non-relativistic matter $\rightarrow \rho \propto a^{-3};\ a \propto (t - t_i)^{2/3}$

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From the second Friedmann eq. (12) one gets more in general:

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\rho(1 + 3w)$$

- deceleration $(\ddot{a} < 0) \Rightarrow w > -1/3$
- acceleration $(\ddot{a} > 0) \Rightarrow w < -1/3$

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1) The flatness problem and primordial inflation

The first Friedmann equation (11) can be recast in the form:

$$\Omega_M + \Omega_K = 1 \quad \text{where} \tag{24}$$

$$\Omega_M = \rho/\rho_{crit}; \; \Omega_K = -K/(aH)^2; \; \rho_{crit} \equiv 3H^2/(8\pi G)$$

for $\ddot{a} < 0$ the curvature term $|\Omega_K|$ increases in time unless $K = 0$

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Observations constrain $|\Omega_K^{(0)}| < 0.008$, which implies a phase of cosmic acceleration in the past (inflation) to reduce the curvature, unless the Universe was extremely close to flatness from the start.

Who cares about acceleration?

Since all known types of matter imply a decelerated expansion, why should we consider the possibility of cosmic acceleration?

1) The flatness problem and primordial inflation

The first Friedmann equation (11) can be recast in the form:

$$\Omega_M + \Omega_K = 1 \quad \text{where} \tag{24}$$

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2) Observational evidence of cosmic acceleration at low redshift: the Dark Energy problem

OBSERVATIONAL EVIDENCE OF COSMIC ACCELERATION: GEOMETRICAL PROBES

OBSERVATIONAL EVIDENCE OF COSMIC ACCELERATION: GEOMETRICAL PROBES (UNDER THE ASSUMPTION OF THE CP)

Age of the Universe (I)

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Combining the first Friedmann eq. and the continuity eq. one has:

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_x) - \frac{K}{a^2} = H_0^2 E^2(z) \quad (25)$$

with

$$E(z) \equiv \left[\Omega_r^{(0)}(1+z)^4 + \Omega_m^{(0)}(1+z)^3 + \Omega_K^{(0)}(1+z)^2 + \Omega_x^{(0)} e^{\int_0^z \frac{3(1+w_x(\tilde{z}))}{1+\tilde{z}} d\tilde{z}}\right]^{1/2} \quad (26)$$

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Now, using the relation $dt = -dz/\left[H(1+z)\right]$ one can compute the age of the Universe:

$$t_0 = \frac{1}{H_0}\int_0^\infty \frac{dz}{E(z)(1+z)} \tag{27}$$

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For a flat $(|\Omega_K| = 0)$ Universe with only relativistic and non-relativistic matter $(i.e.\,\Omega_x = 0)$ one gets:

$$t_0 \simeq \frac{2}{3}H_0^{-1} \quad \approx 10\,\mathrm{Gyr}\ \mathrm{for}\, H_0 \approx 72\ \mathrm{km/s/Mpc} \tag{28}$$

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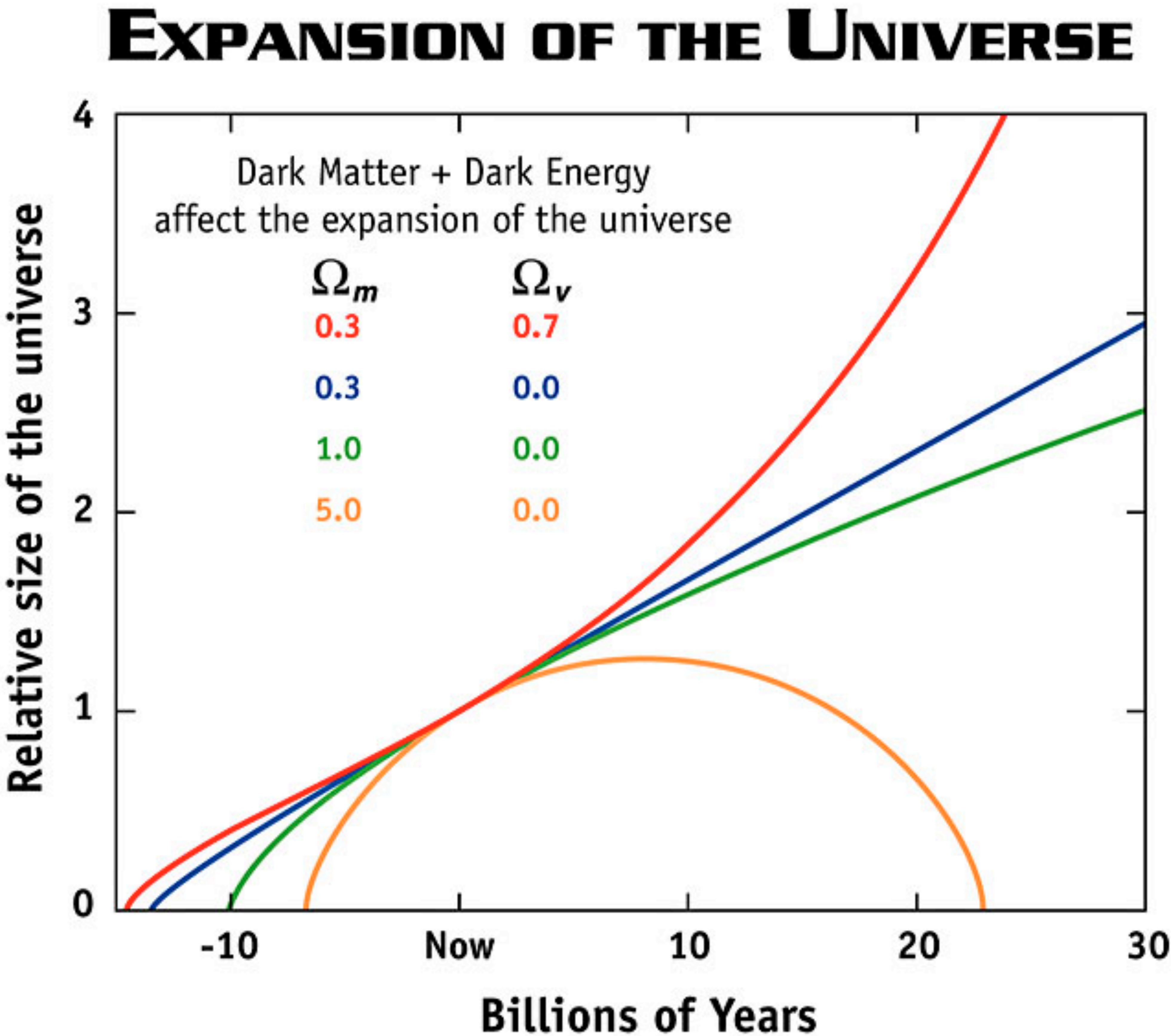
- For a flat Universe with an extra component with $w_x = -1$

$$t_0 \simeq \frac{H_0^{-1}}{3\sqrt{\Omega_x^{(0)}}} \ln \left(\frac{1 + \sqrt{\Omega_x^{(0)}}}{1 - \sqrt{\Omega_x^{(0)}}} \right) \xrightarrow{\Omega_x^{(0)} \longrightarrow 1} \infty \tag{30}$$

Age of the Universe (III)



Type Ia supernovae (I)

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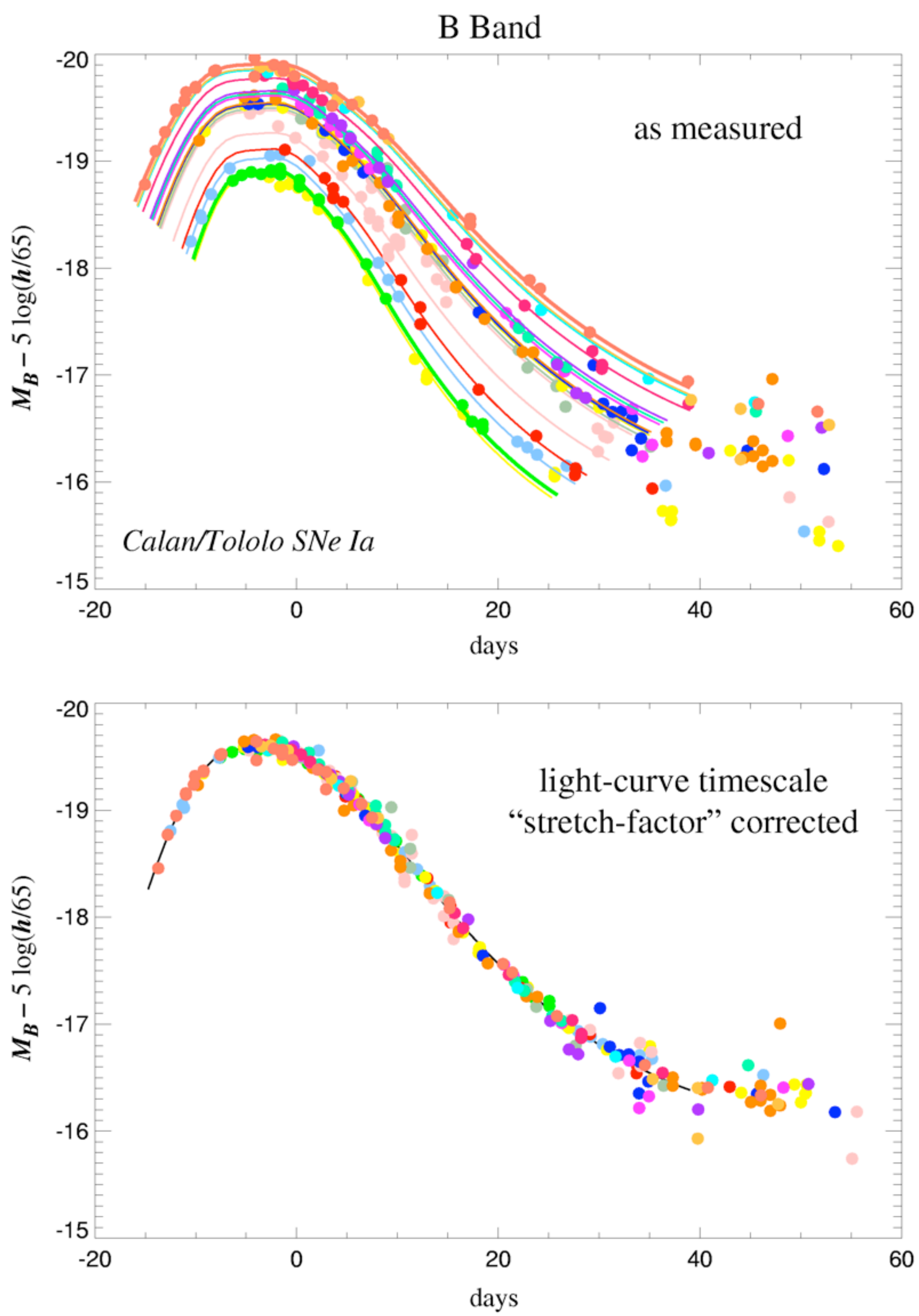
Supernovae of type Ia are astrophysical objects with a “standard” intrinsic luminosity L_s: can be used as “standardiseable candles”



Kim, *et al.* (1997)

Their luminosity distance

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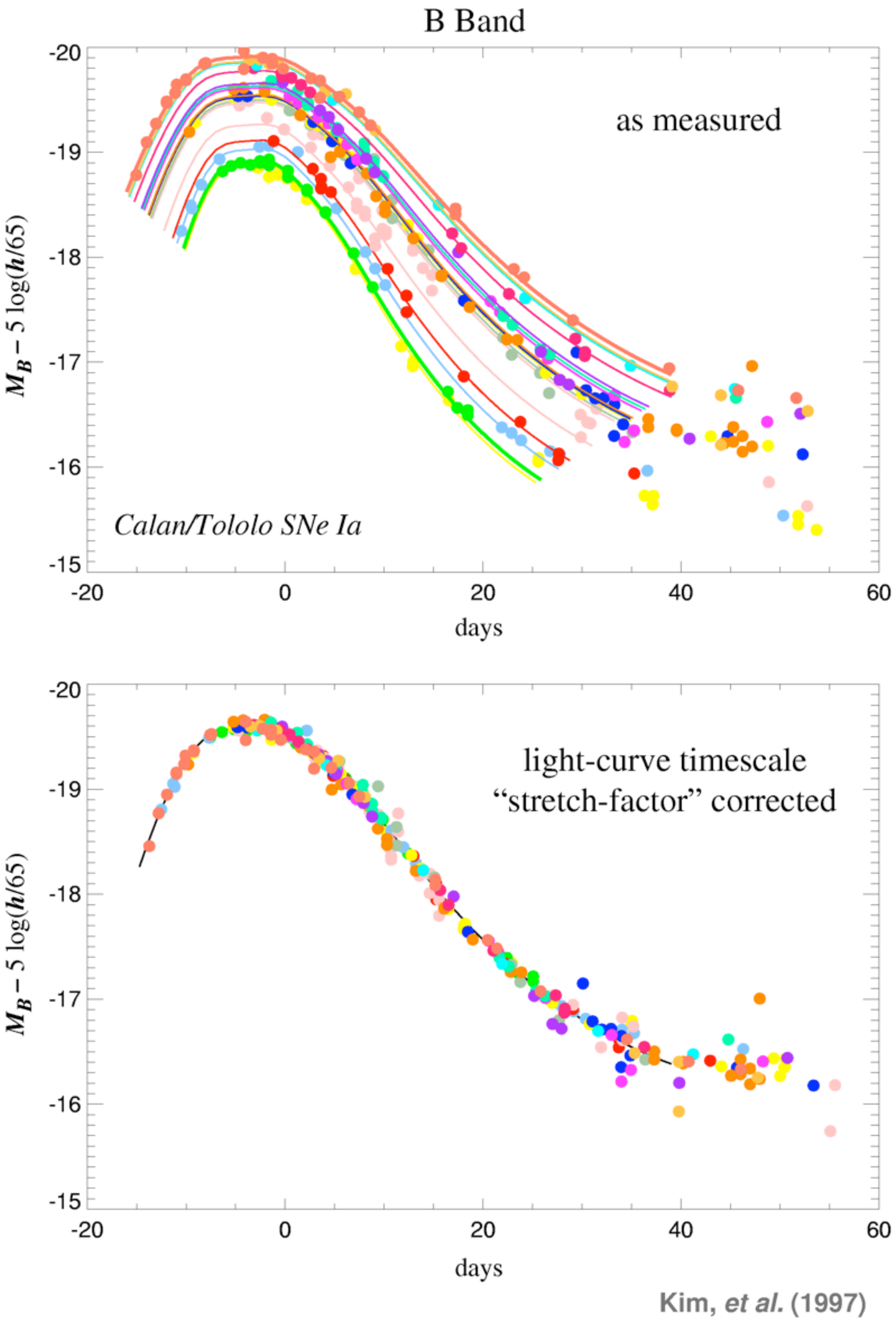

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Since $d_L(z)$ depends on the expansion rate through $E(z)$:

$$d_L = \frac{1+z}{H_0\sqrt{\Omega_K^{(0)}}}\sinh\left(\sqrt{\Omega_K^{(0)}}\int_0^z \frac{d\tilde{z}}{E(\tilde{z})}\right) \tag{32}$$

one can use SnIa measurements to constrain the expansion rate

Type Ia supernovae (II)

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At low redshifts, and for the case of a 3-component Universe with $w_x = \mathrm{const.}$ the luminosity distance can be expanded as:

$$d_L(z) = \frac{1}{H_0}\left[z + \frac{1}{4}\left(1 - 3w_x\Omega_x^{(0)} + \Omega_K^{(0)}\right)z^2 + \mathcal{O}(z^3)\right] \quad (33)$$

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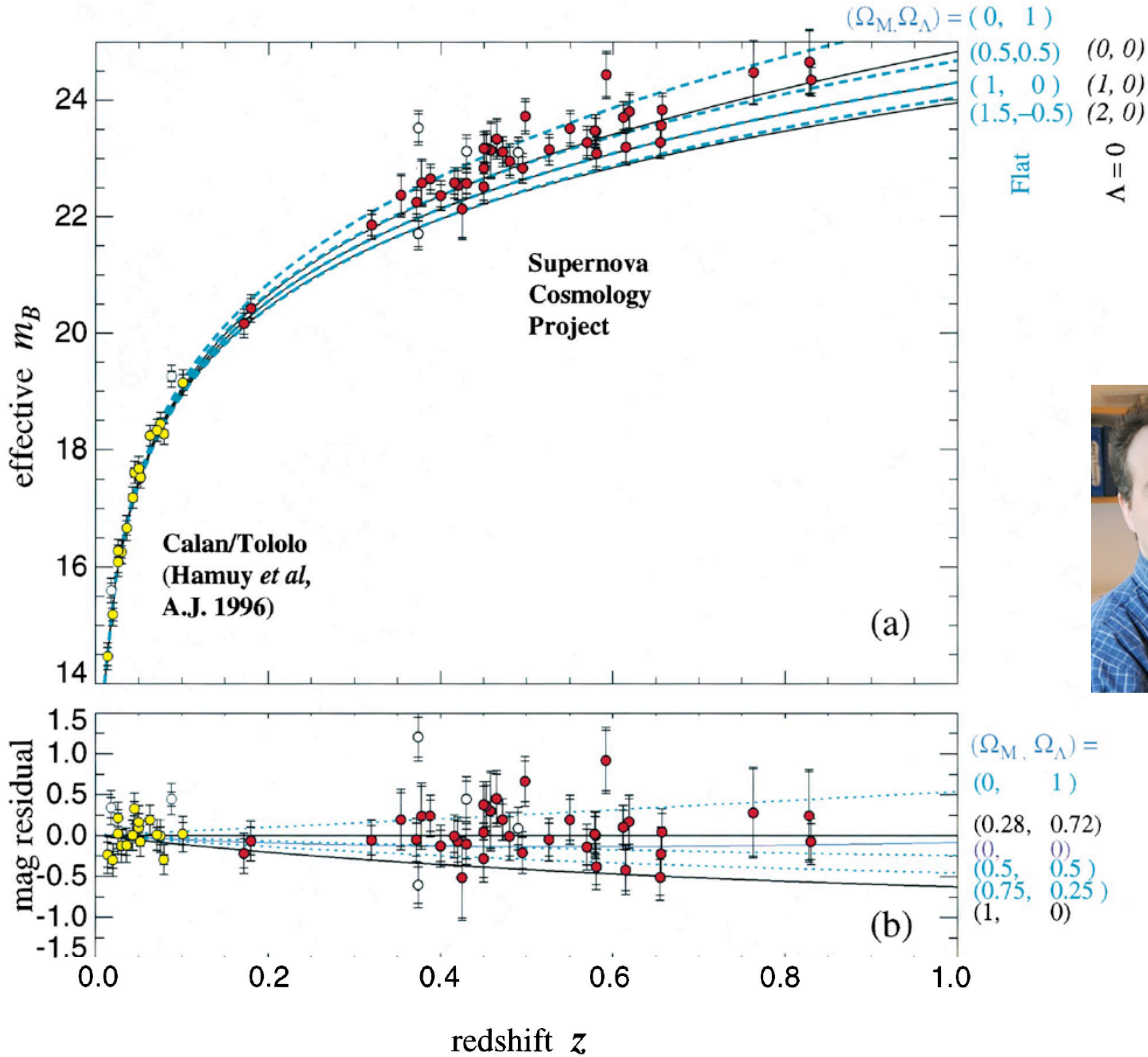
- The luminosity distance is related to the apparent magnitude m:

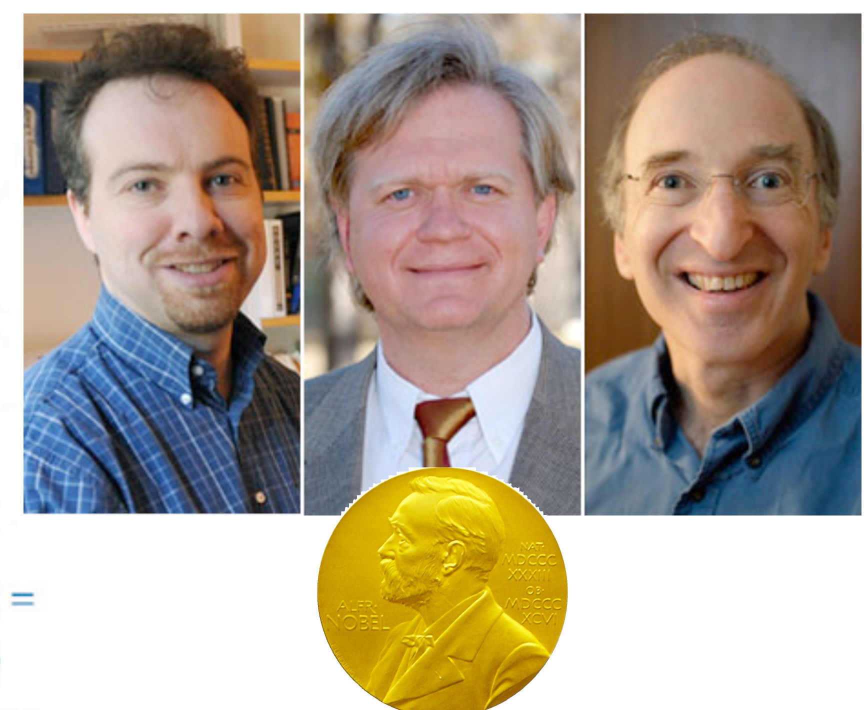
$$m(z) = 5\log_{10} d_L(z) + \mathrm{const.} \quad (35)$$

so the magnitude-redshift diagram can place constraints on $w_x\Omega_x$

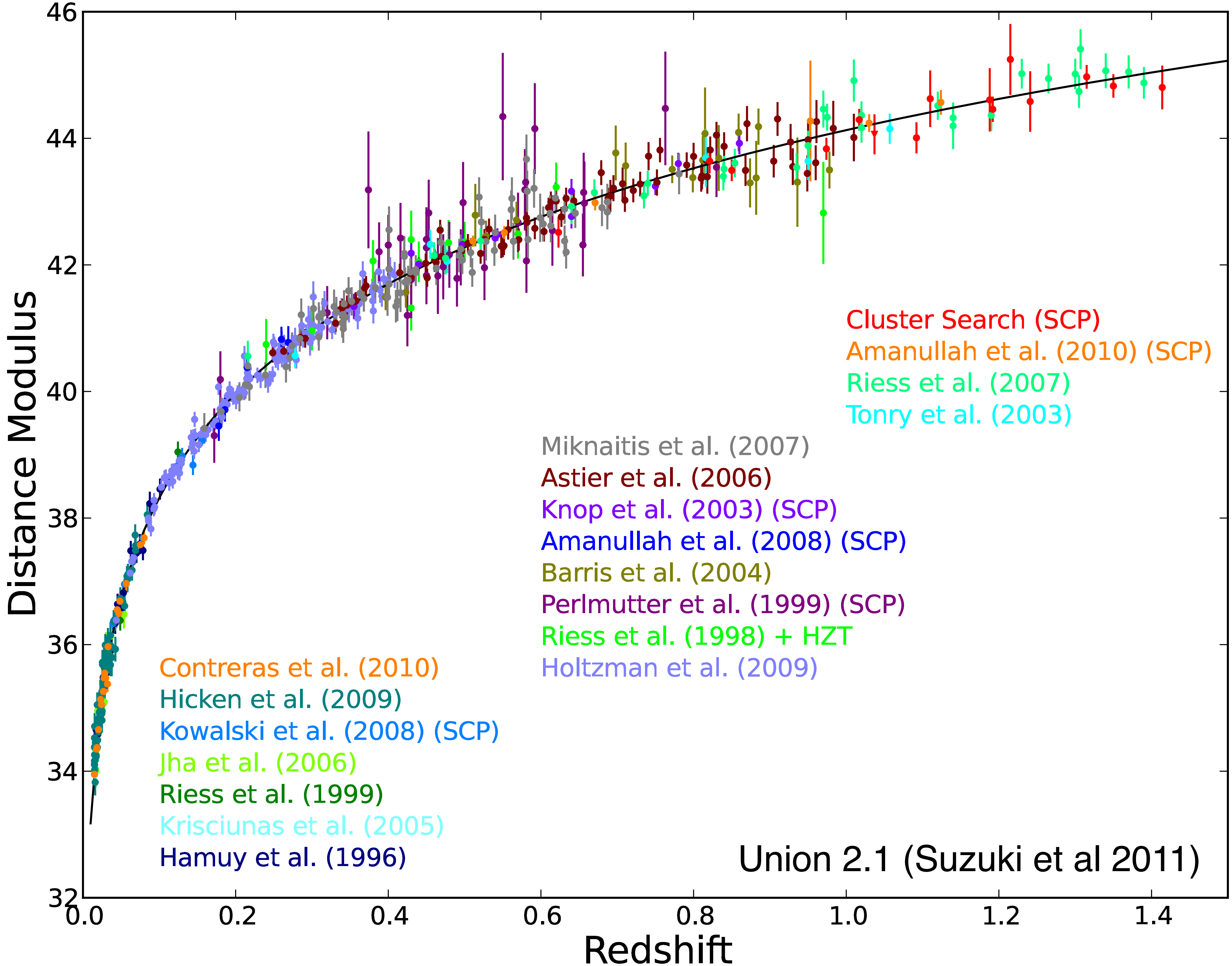
Type Ia supernovae (III)



Perlmutter et al. 1999
Riess et al. 1998
Schmidt et al. 1999



Type Ia supernovae (IV)



THE COSMOLOGICAL CONSTANT

The cosmological constant (I)

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The simplest DE candidate is the cosmological constant Λ, defined by the property: $w_{DE} = w_\Lambda = -1 \Rightarrow \rho_\Lambda = \mathrm{const.}$

The cosmological constant was first introduced by Einstein in 1917 to obtain static solutions to GR equations applied to a model Universe:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow \begin{cases} H^2 = \dfrac{8\pi G}{3}\rho - \dfrac{K}{a^2} + \dfrac{\Lambda}{3} & (36) \\[2ex] \dfrac{\ddot{a}}{a} = -\dfrac{4\pi G}{3}(\rho + 3p) + \dfrac{\Lambda}{3} & (37) \end{cases}$$

for pressureless matter $\ddot{a} = \dot{a} = 0 \Rightarrow \rho = \dfrac{\Lambda}{4\pi G}, \; \dfrac{K}{a^2} = \Lambda$

The cosmological constant (II)

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The Einstein equations with the cosmological constant term can be derived from the most general second order Action in the metric tensor $g_{\mu\nu}$:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda) + S_m \tag{38}$$

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where δS_m defines the matter energy-momentum tensor:

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so that through the Action Principle one gets

$$\delta S = 0 \Rightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{41}$$

The fine-tuning problem of Λ (I)

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In order to explain the observed acceleration the cosmological constant should be of the order:

$$\Lambda \approx H_0^2 \Rightarrow \rho_\Lambda \approx 10^{-123} M_{\rm Pl}^4 \tag{42}$$

The vacuum energy of quantum fields has the property $\rho_{\rm vac} \sim {\rm c}$ so it would be a natural candidate for Λ.

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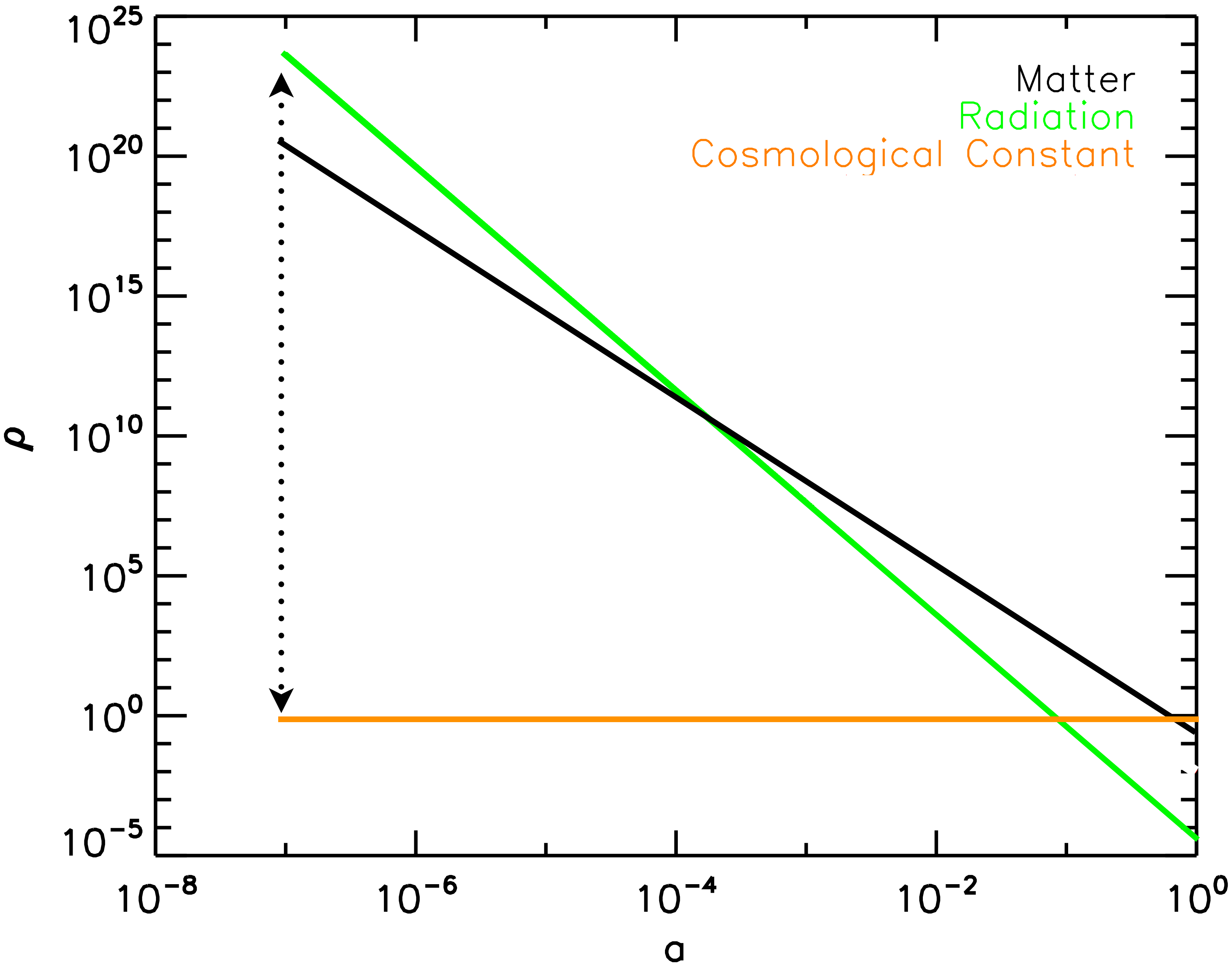
However, from quantum mechanics the zero-point energy of a field of mass m and momentum k is $E = \sqrt{k^2 + m^2}/2$ so that:

$$\rho_{\rm vac} = \int_0^{k_{\rm max}} \frac{d^3k}{(2\pi)^3} \frac{1}{2}\sqrt{k^2+m^2} \approx \frac{k_{\rm max}^4}{16\pi^2} = \frac{M_{\rm Pl}^4}{16\pi^2} \;\; \text{if} \;\; k_{\rm max} = M_{\rm Pl} \quad (43)$$

The fine-tuning problem of Λ (II)

Even if one assumed that $\rho_{\rm vac} = 0$ (for some unknown symmetry principle) and that Λ arises from some other mechanism, still its value has to be fine-tuned at early times due to $\rho_\Lambda = \rm const.$



The coincidence problem of Λ (I)

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Another way to see the coincidence problem is the unlikely coincidence of the crossover time between Ω_Λ and $\Omega_{\rm M}$ with the present time:

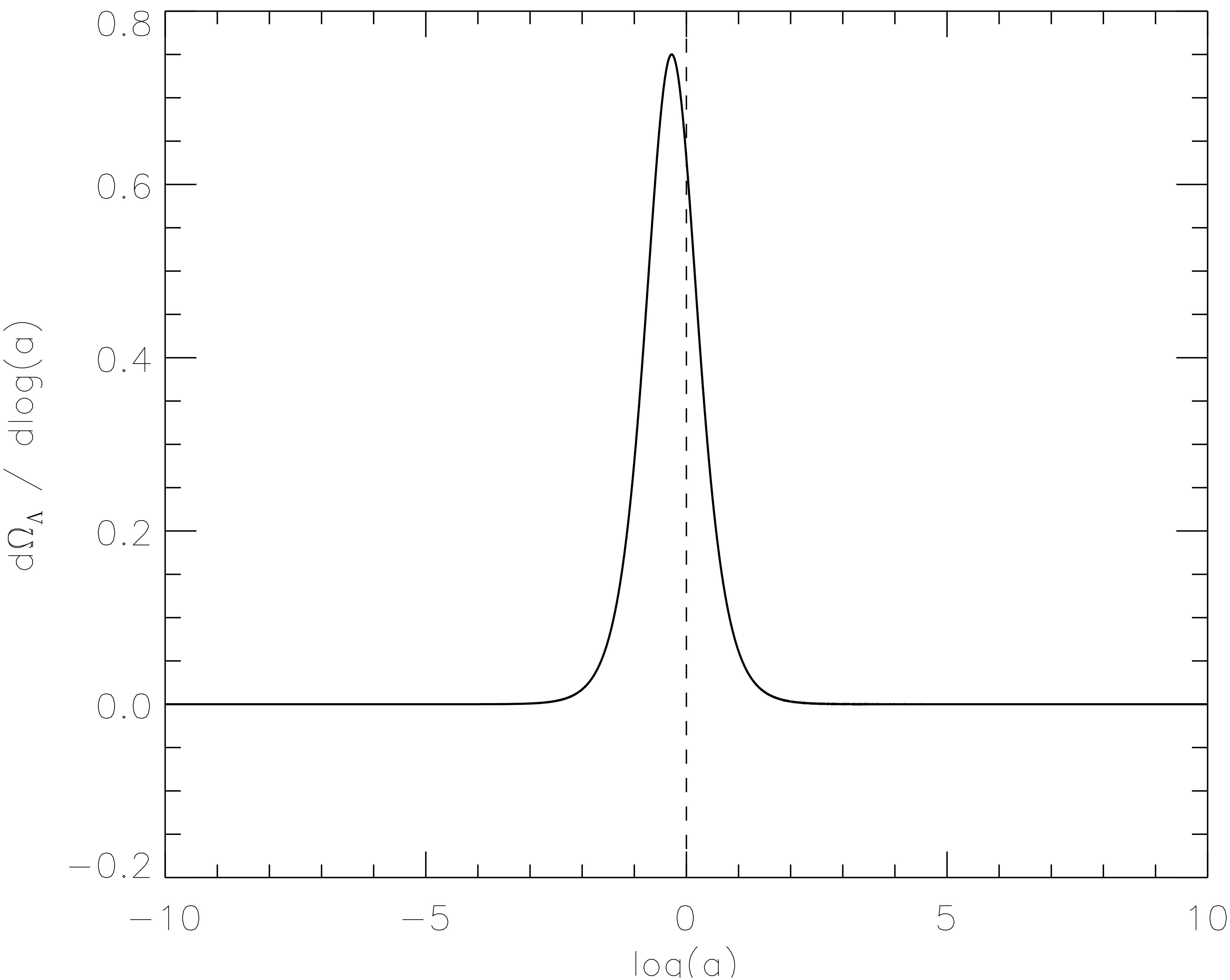
$$z_{\rm cr.} \approx \left(\frac{\Omega_\Lambda^{(0)}}{1-\Omega_\Lambda^{(0)}}\right)^{1/3} - 1 \approx 0.3$$

$$z > z_{\rm cr.} \Rightarrow \Omega_\Lambda \ll \Omega_{\rm M}$$

$$z < z_{\rm cr.} \Rightarrow \Omega_\Lambda \gg \Omega_{\rm M}$$

so that the present appears to be a special and unique time in the cosmic evolution.

The coincidence problem of Λ (II)



Recap Lecture 1

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General Relativity

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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result in the Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

that fully describe the dynamics of the homogeneous universe.

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Several observations (age of the Universe, type Ia Supernovae) are inconsistent with a decelerated expansion. A possible explanation is a cosmological constant:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

but this suffers of fundamental theoretical problems (fine-tuning, coincidence)