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COSMIC ACCELERATION: FROM THE COSMOLOGICAL CONSTANT TO DARK ENERGY AND MODIFIED GRAVITY THEORIES

Astrophysical probes of fundamental physics
ferrara, 7-11 september 2015

◉ Basics of structure formation

- *Deviations from homogeneity*
- *Perturbed cosmological equations in the Newtonian gauge*
- *Linear growth of density perturbations*
- *The matter power spectrum*

◉ Observational evidence of Dark Energy from perturbations

- Cosmic Microwave Background: the ISW effect
- Angular correlation function of galaxies

◉ Homogeneous DE models beyond the cosmological constant

- Dark Energy parameterisations
- Early Dark Energy
- Scalar field models: Quintessence and k-essence

◉ Interacting Dark Energy and Modified Gravity theories

- Coupled Quintessence
- *f(R)* gravity

Basics of structure formation

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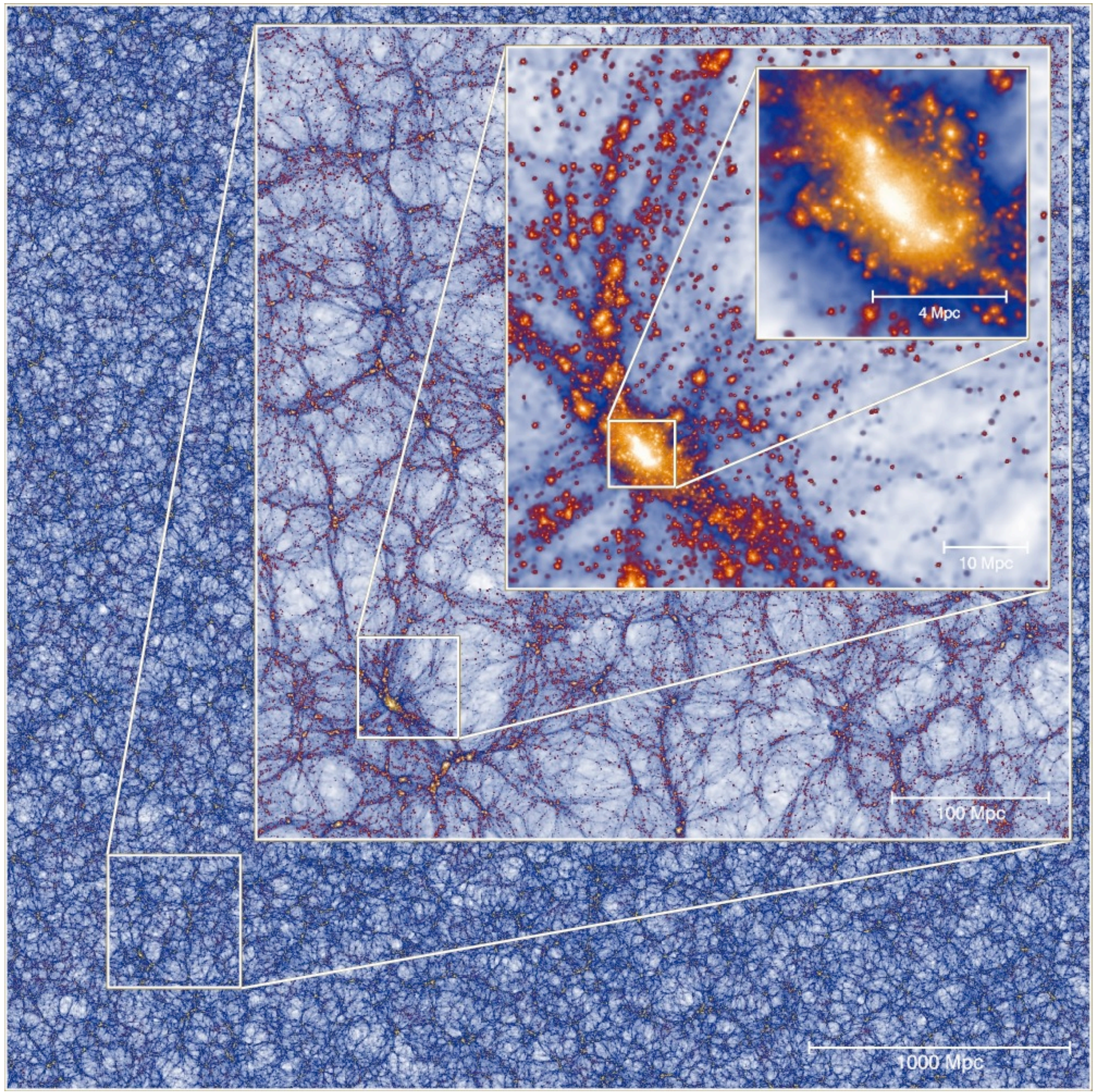

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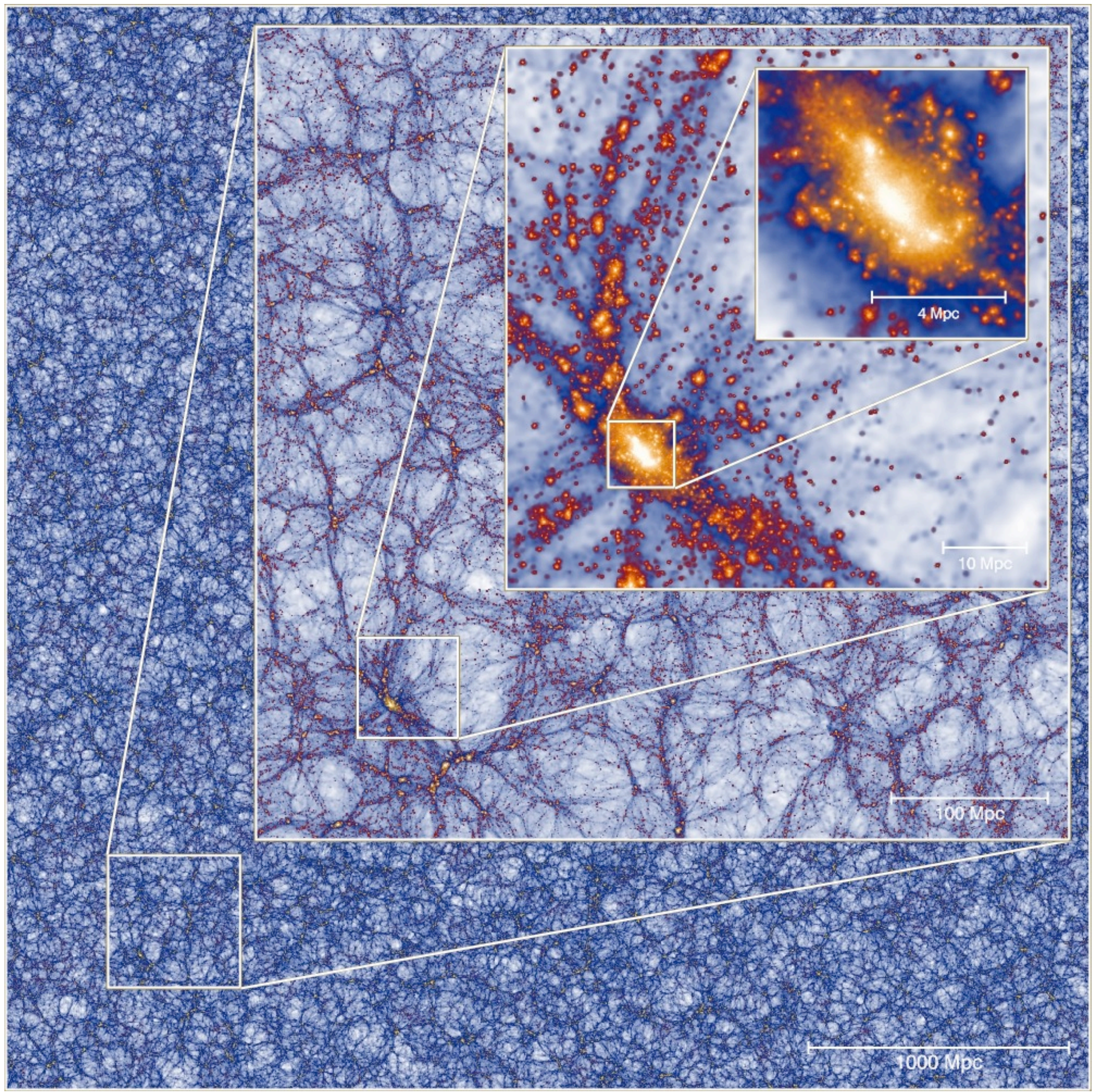

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To quantify the inhomogeneity of a distribution we use the 2-point correlation function ξ:

$$\xi(r) = \frac{\langle \rho(r) \rangle}{\rho_0} - 1 \quad (45)$$

so that

$$\int \xi(r) dV = 0 \quad (46)$$

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To describe an inhomogeneous Universe that recovers homogeneity at large scales we need to derive again GR equations using a "perturbed" FLRW metric (in conformal time $d\eta \equiv dt/a$):

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where (by choosing the Newtonian gauge):

$$g^{(0)}_{\mu\nu} = a^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} \end{pmatrix} \qquad \delta g_{\mu\nu} = a^2 \begin{pmatrix} -2\Psi & 0 \\ 0 & 2\Phi\delta_{ij} \end{pmatrix} \tag{48}$$

so that the perturbed line element in conformal time is:

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where Ψ and Φ are perturbation functions and are assumed to be small:

$$\Psi,\, \Phi \ll 1 \tag{50}$$

Deviations from homogeneity (III)

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With the perturbed FLRW metric in the Newtonian gauge one can derive the perturbed version of the Einstein equations:

$$G^{\mu(0)}_{\nu} + \delta G^{\mu}_{\nu} = 8\pi G(T^{\mu(0)}_{\nu} + \delta T^{\mu}_{\nu}) \quad \Rightarrow \boxed{\delta G^{\mu}_{\nu} = 8\pi G \delta T^{\mu}_{\nu}} \quad (51)$$

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where we have introduced the density contrast δ and the sound speed c_s^2:

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&(0,i) : \nabla^2(\Phi' - \mathcal{H}\Psi) = 4\pi G a^2 (1+w)\rho\theta \\
&(i,j) : \Psi = -\Phi \\
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The (i,j) component comes from the assumption of a perturbed perfect fluid $\delta T^i_j = 0$, and the (i,i) component becomes a dynamic equation for the (only) gravitational potential Φ

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These equations strongly simplify for the case of non-relativistic matter ($w \simeq 0, c_s \simeq 0$) at small scales ($\Phi' \simeq 0$):

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Finally, from Einstein (0,0) at small scales ($\mathcal{H}^2\Phi \ll \nabla^2\Phi$) one gets the sub-horizon Poisson equation:

$$\boxed{\nabla^2\Phi = 4\pi G a^2 \rho\delta = \frac{3}{2}\frac{8\pi G\rho}{3H^2}a^2H^2\delta = \frac{3}{2}\Omega\mathcal{H}^2\delta} \tag{58}$$

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For the case of two pressureless fluids (as e.g. baryons and dark matter), the gravitational instability equation is easily generalised:

$$\delta_c'' + \mathcal{H}\delta_c' - \frac{3}{2}\mathcal{H}^2(\Omega_c\delta_c + \Omega_b\delta_b) = 0 \quad (61)$$

$$\delta_b'' + \mathcal{H}\delta_b' - \frac{3}{2}\mathcal{H}^2(\Omega_c\delta_c + \Omega_b\delta_b) = 0 \quad (62)$$

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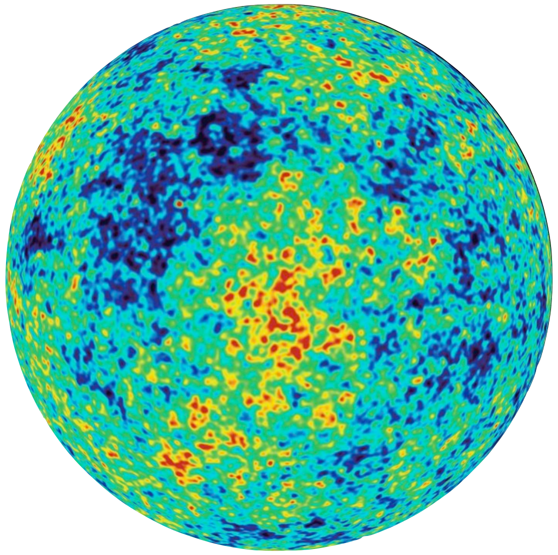
Primordial density field

$z_{\rm CMB} \approx 10^3, a_{\rm CMB} \approx 10^{-3}$



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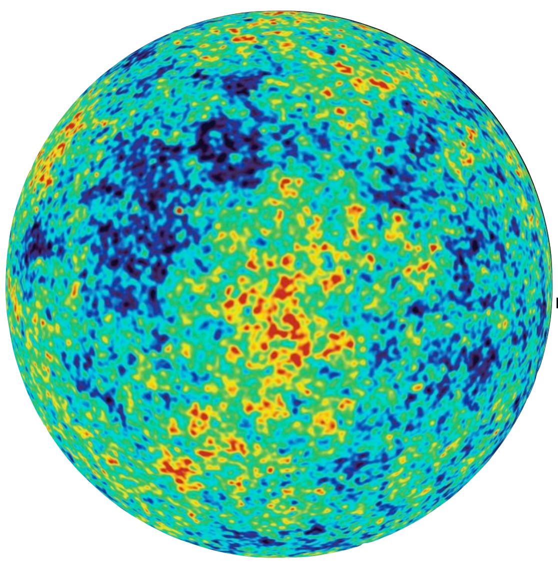

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Structures in the present-day Universe

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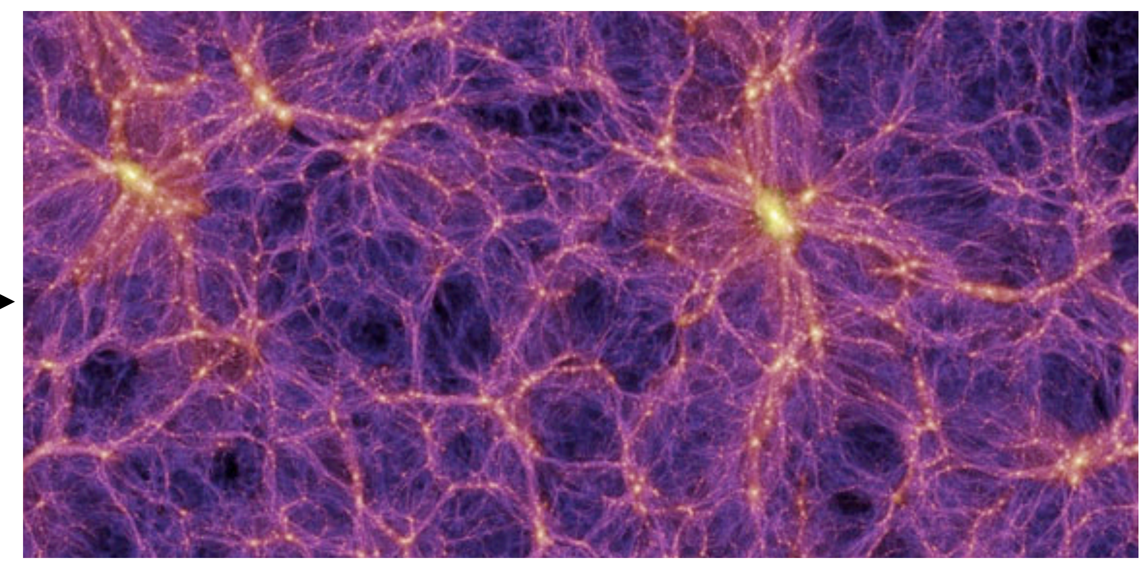

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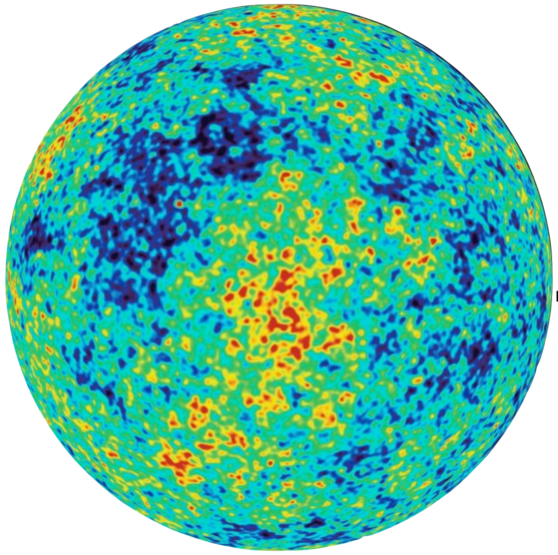

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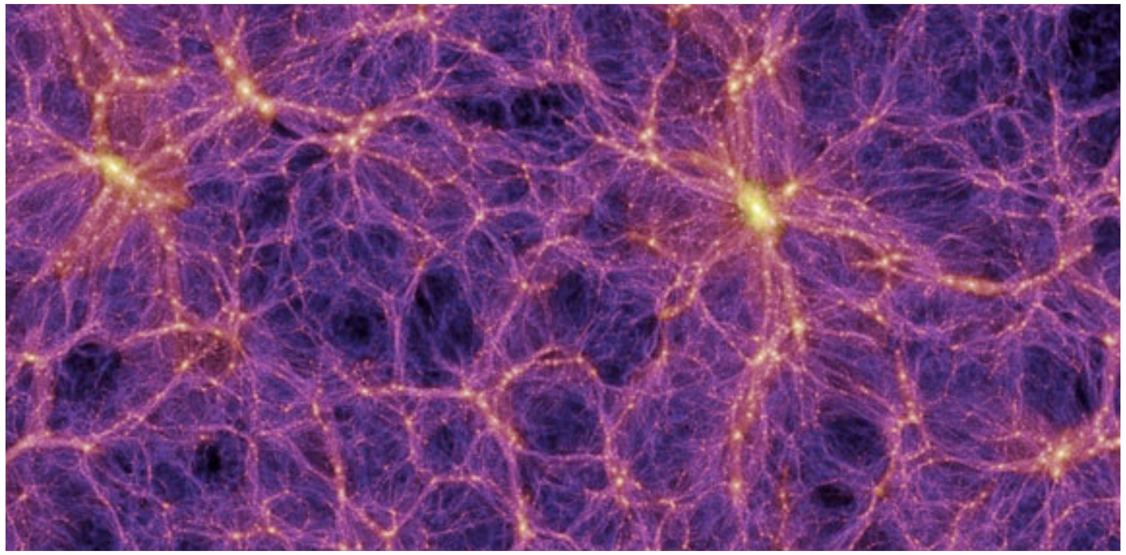

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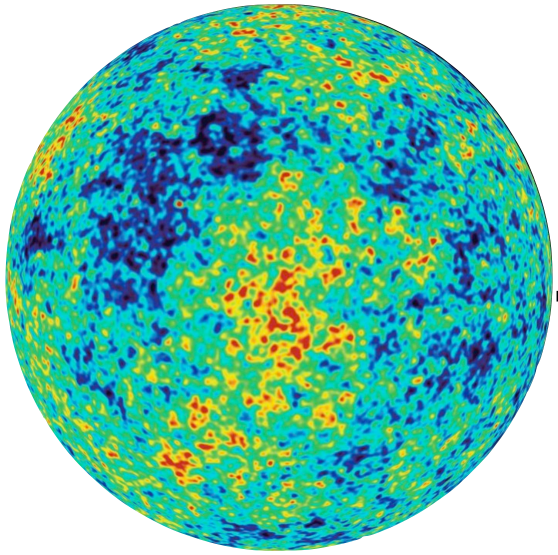

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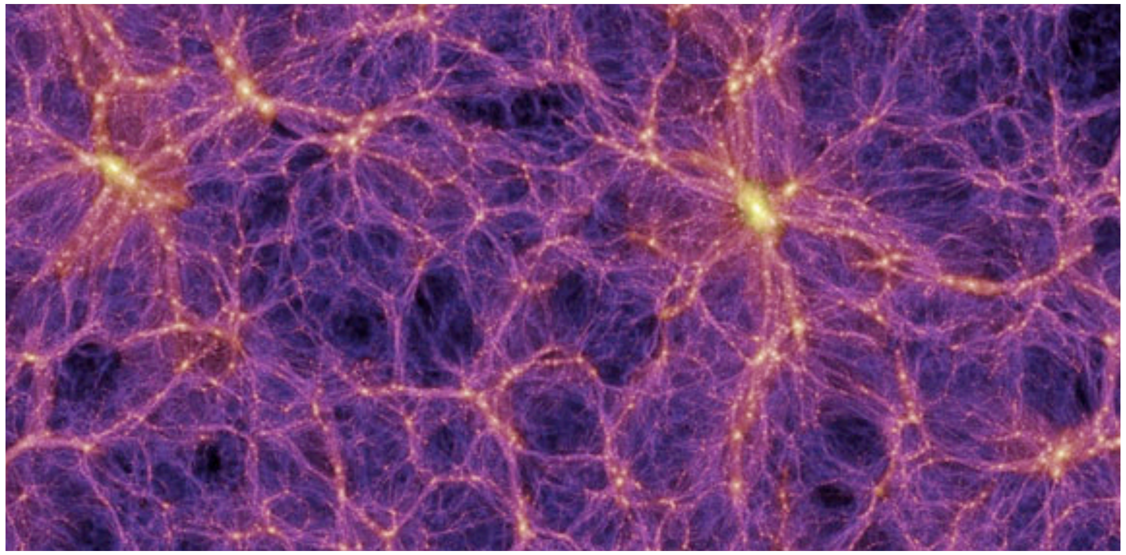

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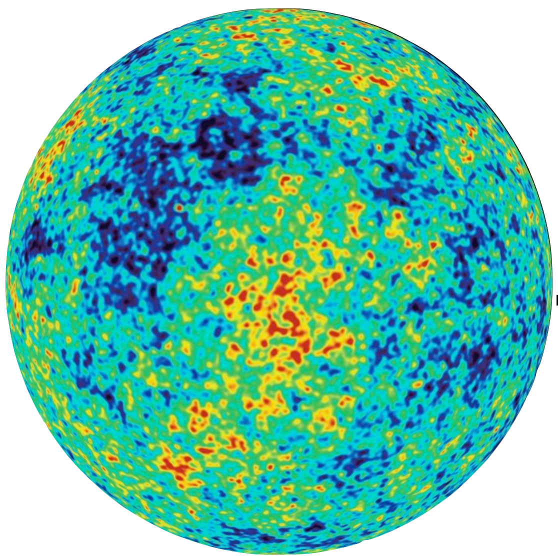

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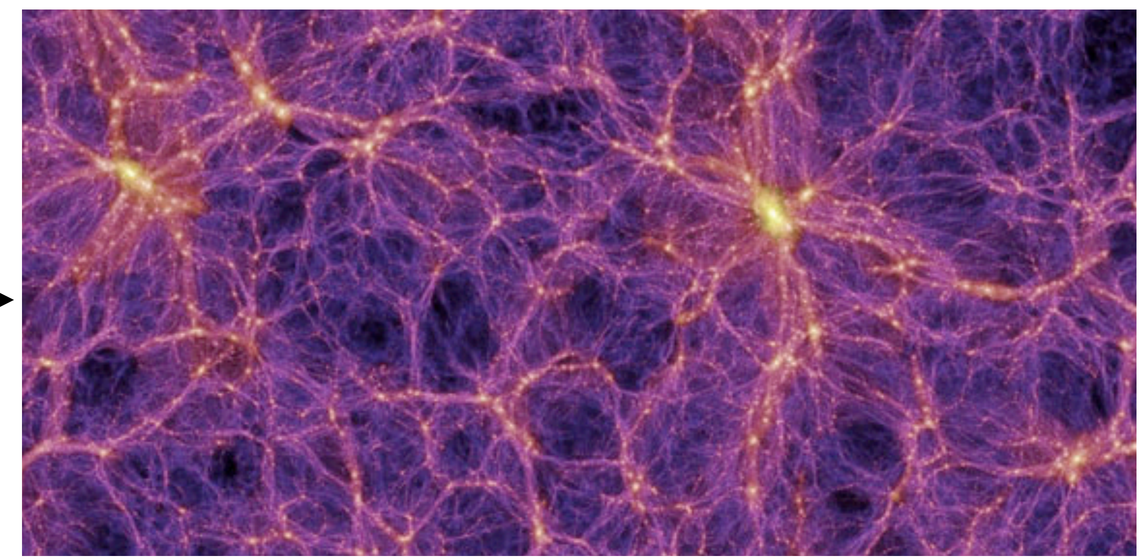

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With more complicated Dark Energy models also $\mathcal{H}$ changes non-trivially, and additional forces might come to play, so that $m \lessgtr 1$

The matter power spectrum

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How to measure the level of inhomogeneity of a perturbation field?
A convenient approach is to decompose the field in Fourier modes:

$$\delta_{\mathbf{k}} = \frac{1}{V}\int \delta(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} dV \qquad \langle \delta_{\mathbf{k}} \rangle_V = \langle \delta(\mathbf{x}) \rangle_V = 0 \qquad (63)$$

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The simplest non-trivial statistics is the power spectrum:

$$P(\mathbf{k}) = V \delta_{\mathbf{k}} \delta^*_{\mathbf{k}} \quad \Rightarrow P(\mathbf{k}) = \int \xi(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} dV \tag{64}$$

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One can then compute the variance of the field in spheres of radius R using a spherical top-hat window function $W(\mathbf{x}) = 3/4\pi R^3$ for $\mathbf{x} - \mathbf{x_0} < R$ and $W(\mathbf{x}) = 0$ otherwise:

$$\sigma_R^2 = \frac{1}{2\pi} \int P(k) W^2(k) k^2 dk \tag{65}$$

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When we say that $(\delta\rho/\rho)_0 \approx 1$ what we actually mean is that:

$$\sigma_8 \equiv \sigma_{8h^{-1}\mathrm{Mpc}} \approx 1 \tag{66}$$

OBSERVATIONAL EVIDENCE OF COSMIC ACCELERATION: STRUCTURE FORMATION PROBES

Cosmic Microwave Background

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Dark Energy is a low-redshift phenomenon, so it should not affect the properties of the CMB at the last scattering surface. However, CMB photons travel through a Dark Energy dominated Universe before reaching us, and their properties can be modified by DE.

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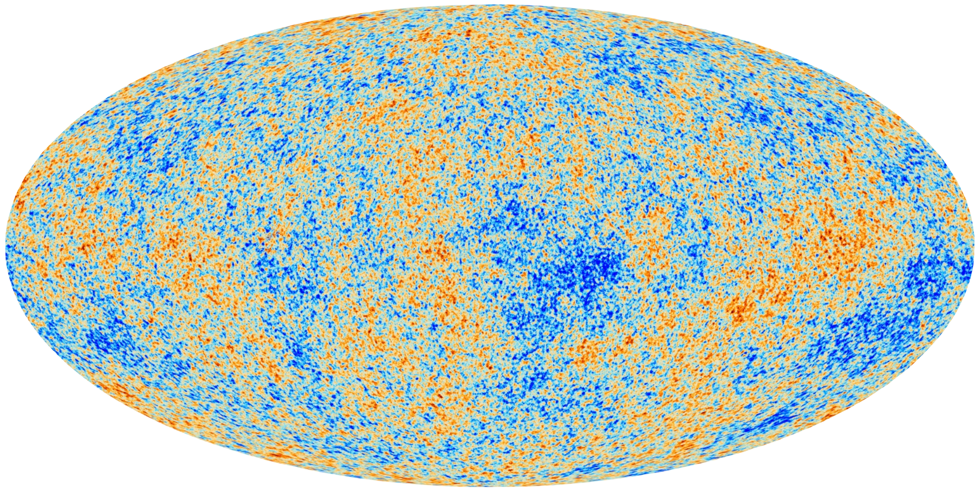

CMB temperature anisotropies (Planck 2015)

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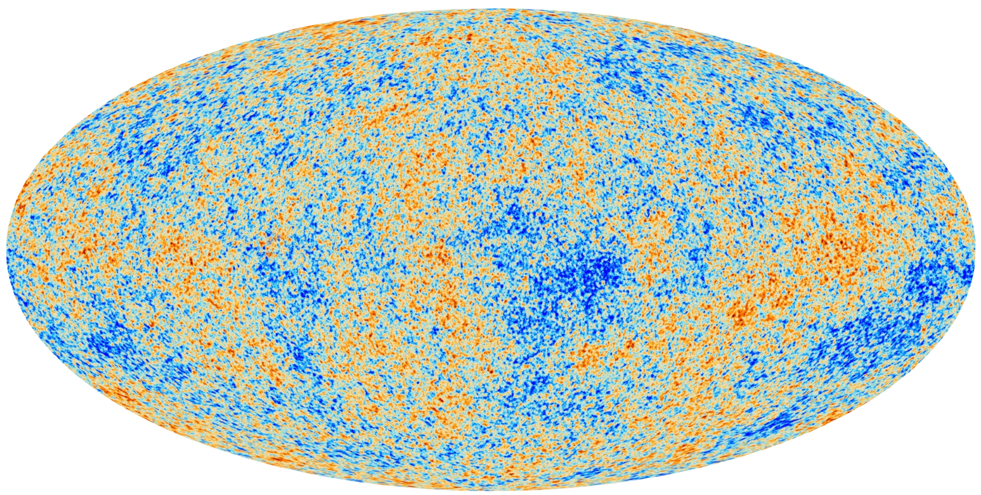

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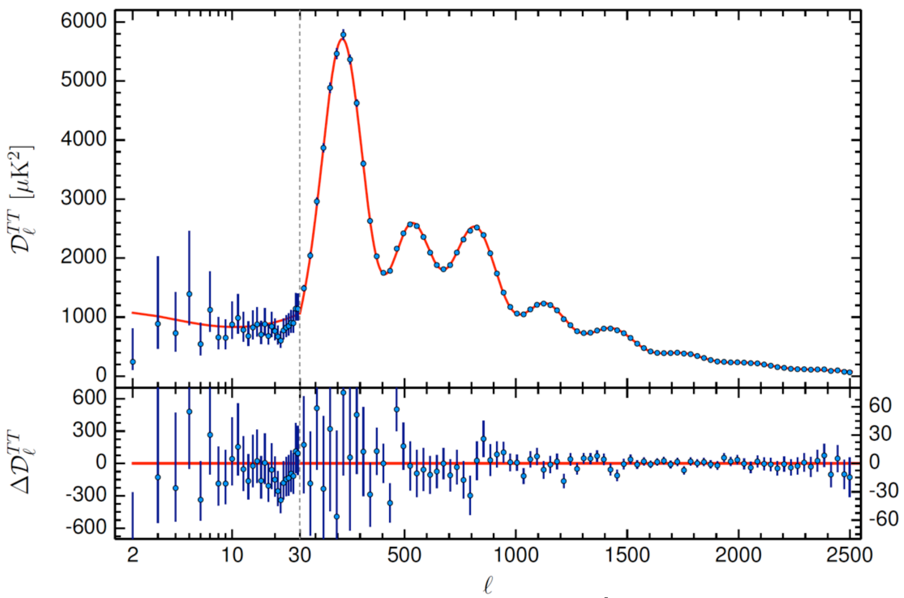

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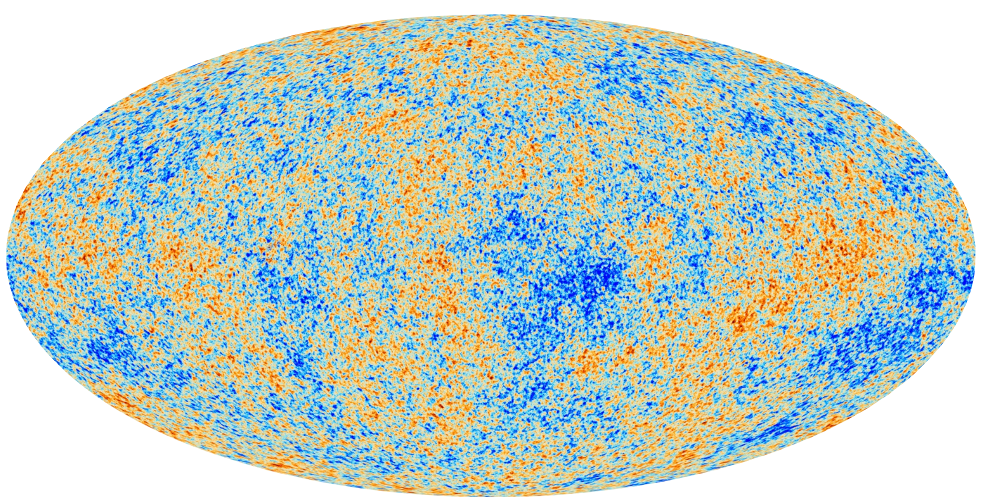

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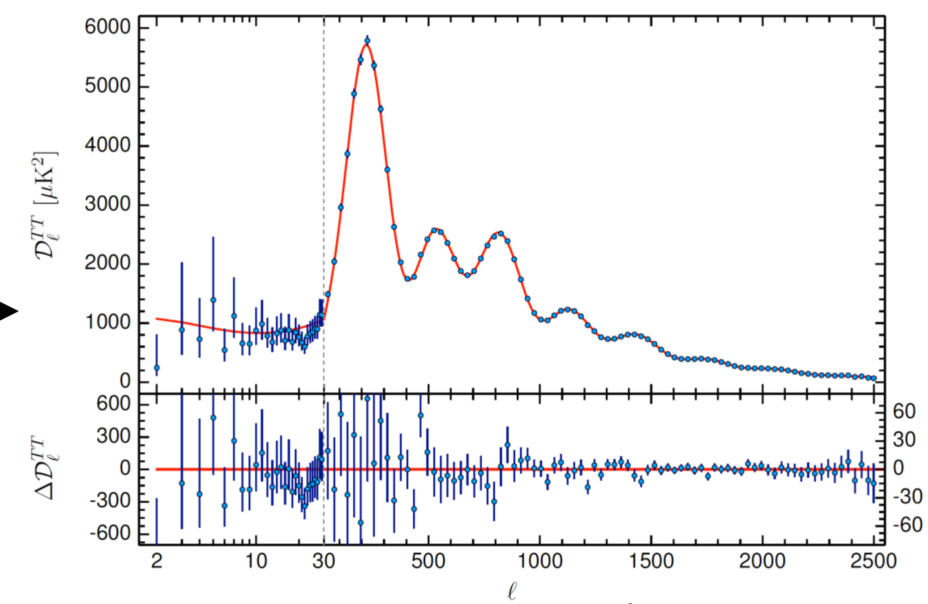

Angular power spectrum (Planck 2015)

In particular, the power at large angles ($\ell \lesssim 30$) is dominated by the ISW contribution:

$$(\delta T/T)_{\rm ISW} \propto \int_E^O 2(\partial\Phi/\partial\eta)d\eta \quad \begin{cases} = 0 & \text{for matter domination} \\ \neq 0 & \text{in the presence of DE} \end{cases} \tag{67}$$

More on this in the CMB lectures (Burigana, Mandolesi, Natoli)

Large Scale Structures

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The cosmological constant and cold dark matter

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Department of Physics, University of Oxford, Oxford OX1 3RH, UK

THE cold dark matter (CDM) model[1–4] for the formation and distribution of galaxies in a universe with exactly the critical density is theoretically appealing and has proved to be durable, but recent work[5–8] suggests that there is more cosmological structure on very large scales ($l > 10\ h^{-1}$ Mpc, where h is the Hubble constant H_0 in units of $100\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$) than simple versions of the CDM theory predict. We argue here that the successes of the CDM theory can be retained and the new observations accommodated in a spatially flat cosmology in which as much as 80% of the critical density is provided by a positive cosmological constant, which is dynamically equivalent to endowing the vacuum with a non-zero energy density. In such a universe, expansion was dominated by CDM until a recent epoch, but is now governed by the cosmological constant. As well as explaining large-scale structure, a cosmological constant can account for the lack of fluctuations in the microwave background and the large number of certain kinds of object found at high redshift.

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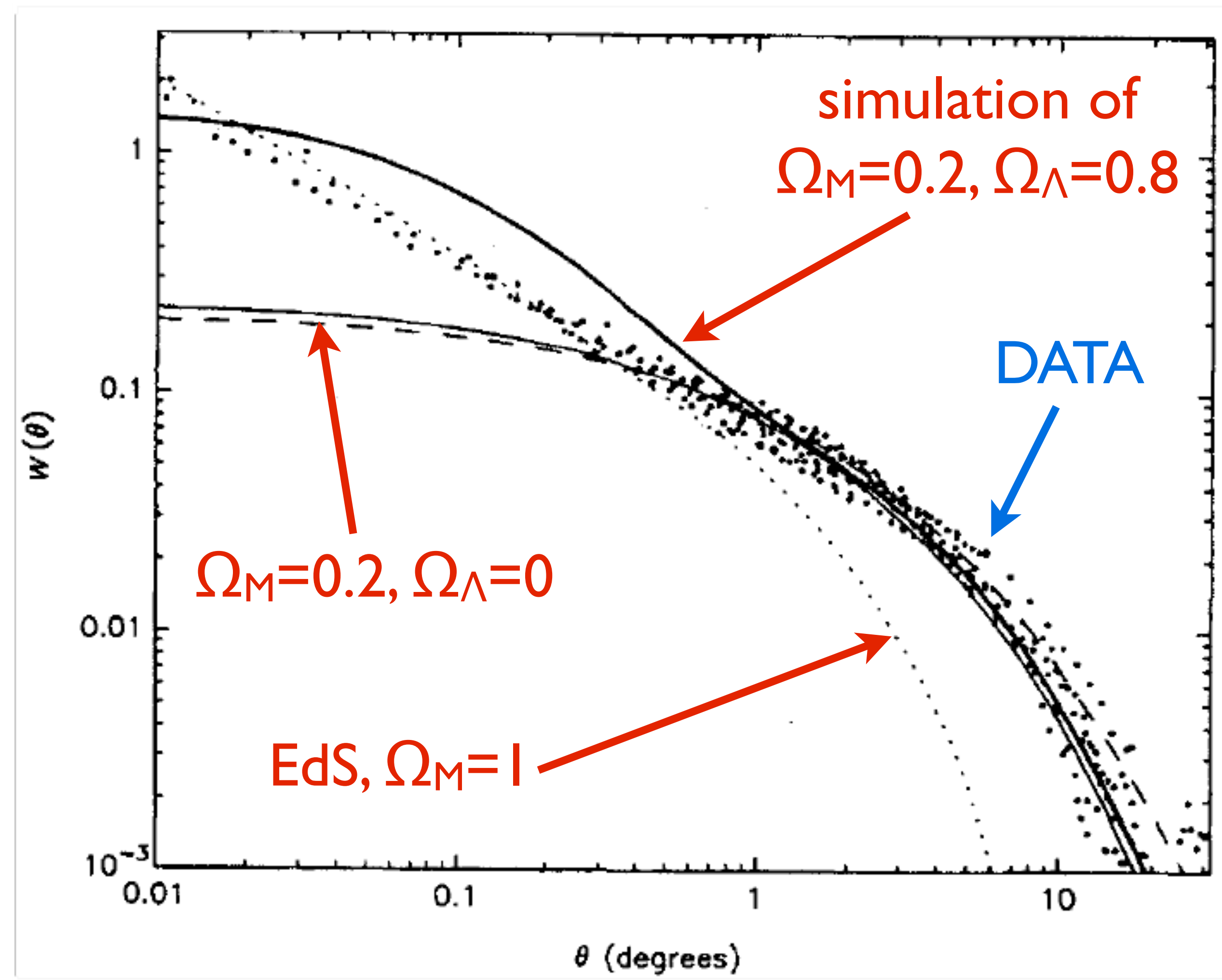

DARK ENERGY MODELS BEYOND THE COSMOLOGICAL CONSTANT

Classification of Dark Energy models

Classification of Dark Energy models

	time evolution	spatial fluctuations	interactions
Λ	✘	✘	✘
Dynamical DE (DE parameterisations, Quintessence, k-essence)	✔ a dynamical (scalar) degree of freedom	✘ no clustering at sub-horizon scales	✘ minimally-coupled to matter fields

Dark Energy parameterisations (I)

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A first step to generalise Dark Energy beyond the cosmological constant is to release the property $w_{\rm DE} = -1 \Rightarrow \rho_{\rm DE} = {\rm const.}$ that characterises Λ, for instance by considering phenomenological cases like $w_{\rm DE} = {\rm const.} < -1/3$ or even $w_{\rm DE}(z)$

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Some popular parameterisations are:

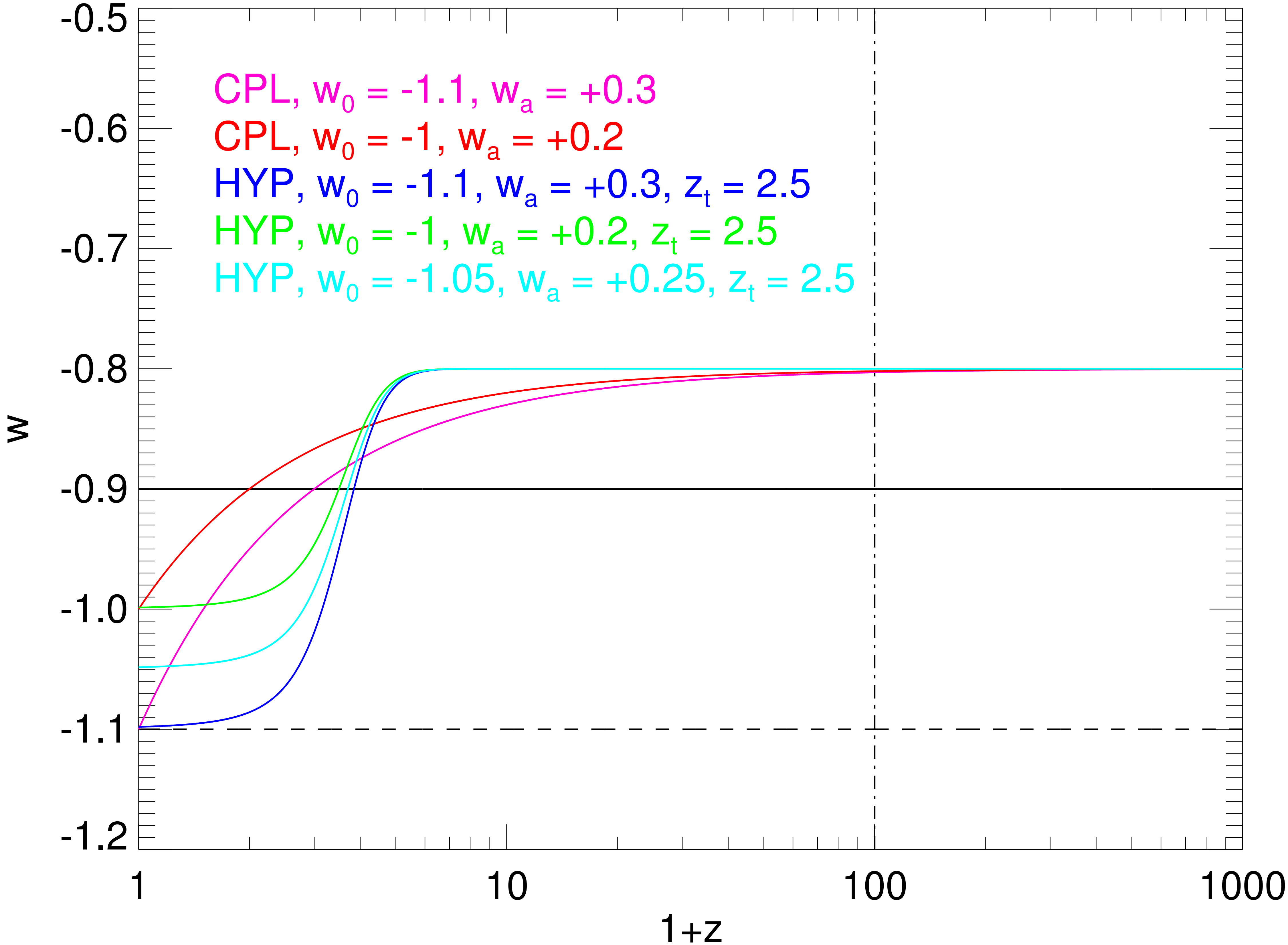
Chevalier-Polarski-Linder (CPL):

$$\boxed{w(a) = w_0 + w_a(1-a)} \tag{68}$$

Early Dark Energy (EDE, Doran & Robbers 2006):

$$\boxed{w_{\rm DE}(a) = \frac{w_0}{1 + b\ln(1/a)}} \quad b = -\frac{3w_0}{\ln\frac{1-\Omega_{\rm EDE}}{\Omega_{\rm EDE}} + \ln\frac{1-\Omega_M}{\Omega_M}} \tag{69}$$

Dark Energy parameterisations (II)



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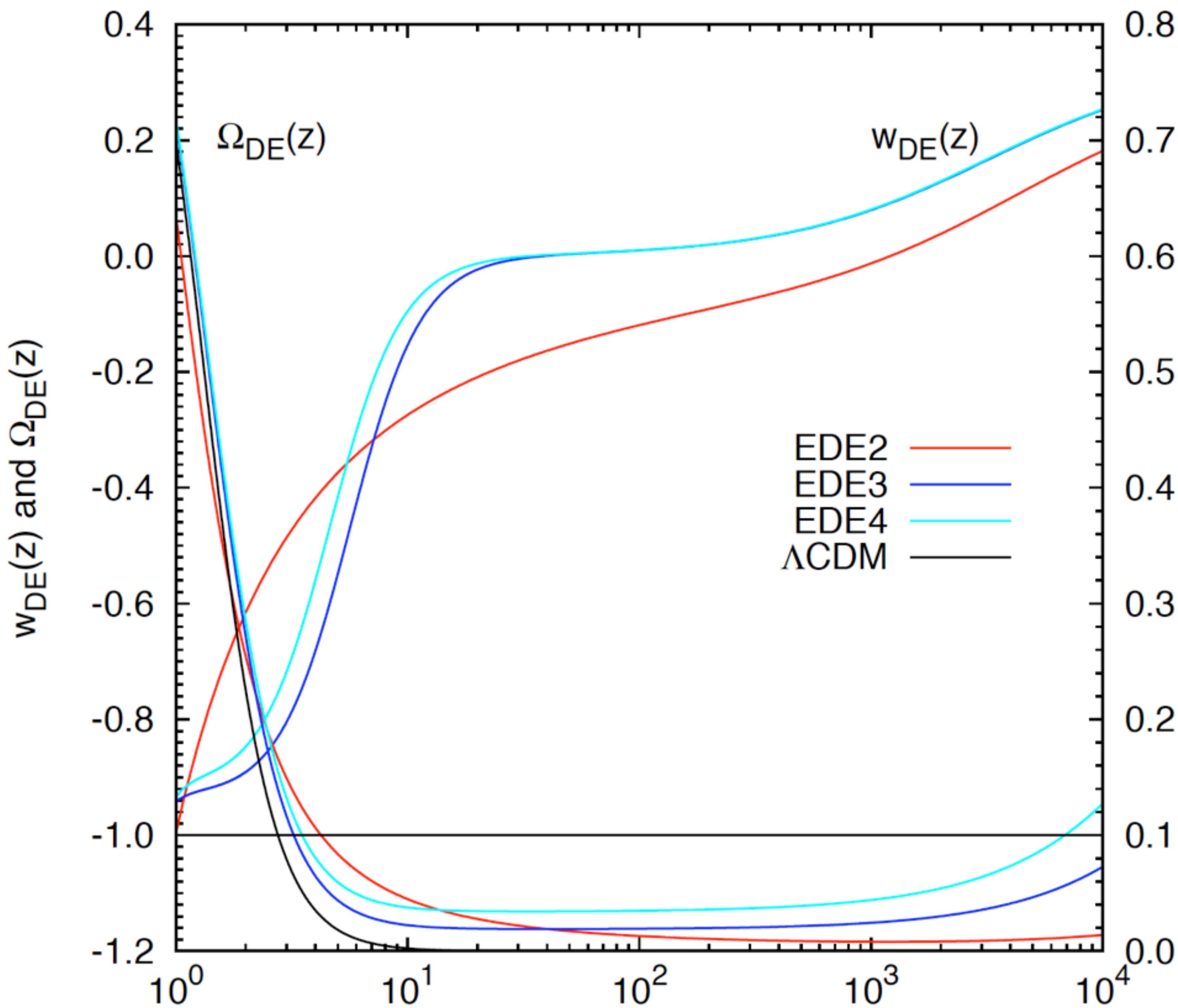

Waizmann & Bartelmann 2009

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A more physically motivated class of dynamical Dark Energy scenarios is given by scalar field models based on a scalar degree of freedom $\phi(t)$ evolving in a self-interaction potential $V(\phi)$:

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \tag{70}$$

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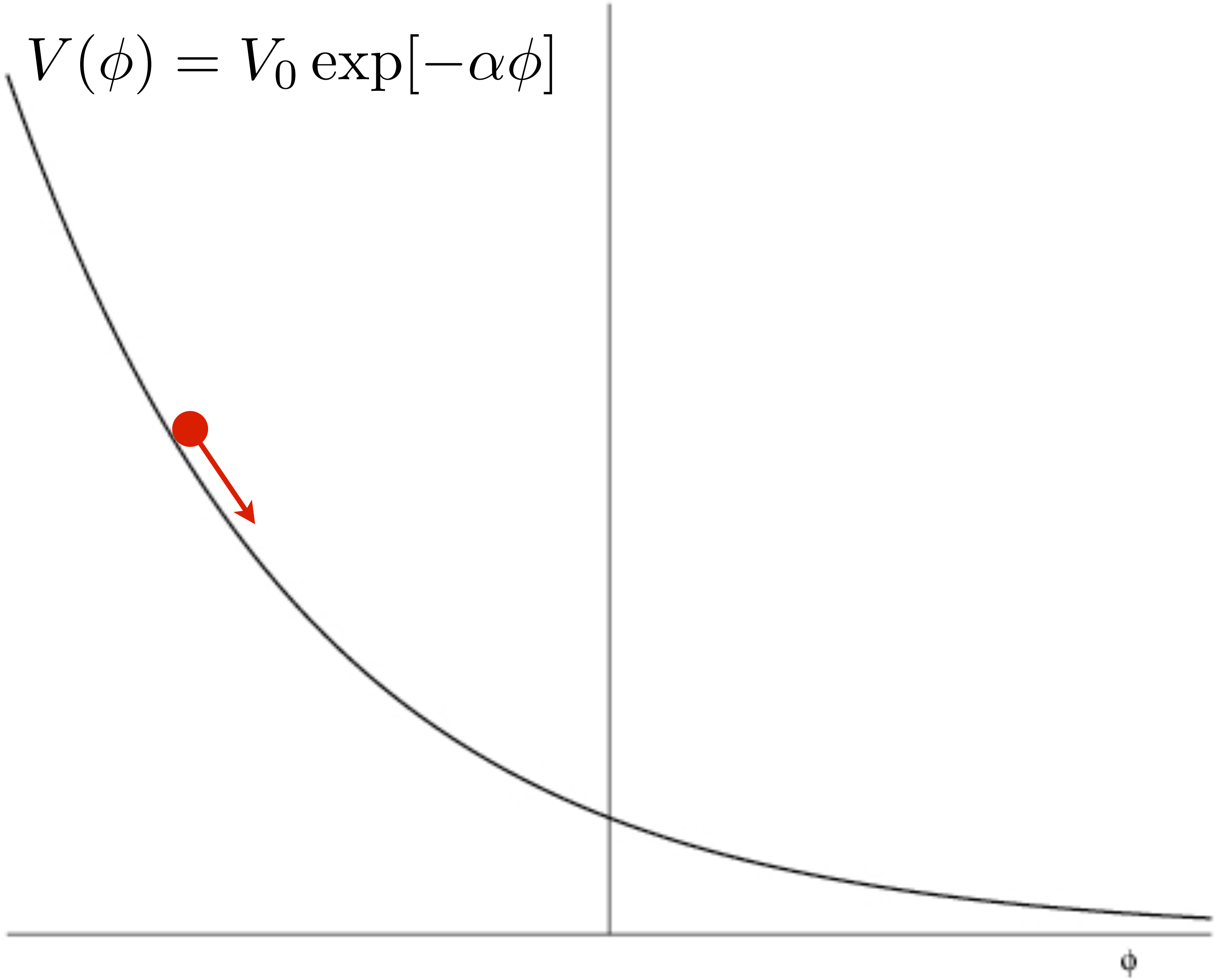
and a dynamical equation for the field (Klein-Gordon equation):

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0 \Rightarrow \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{73}$$

Quintessence models (II)



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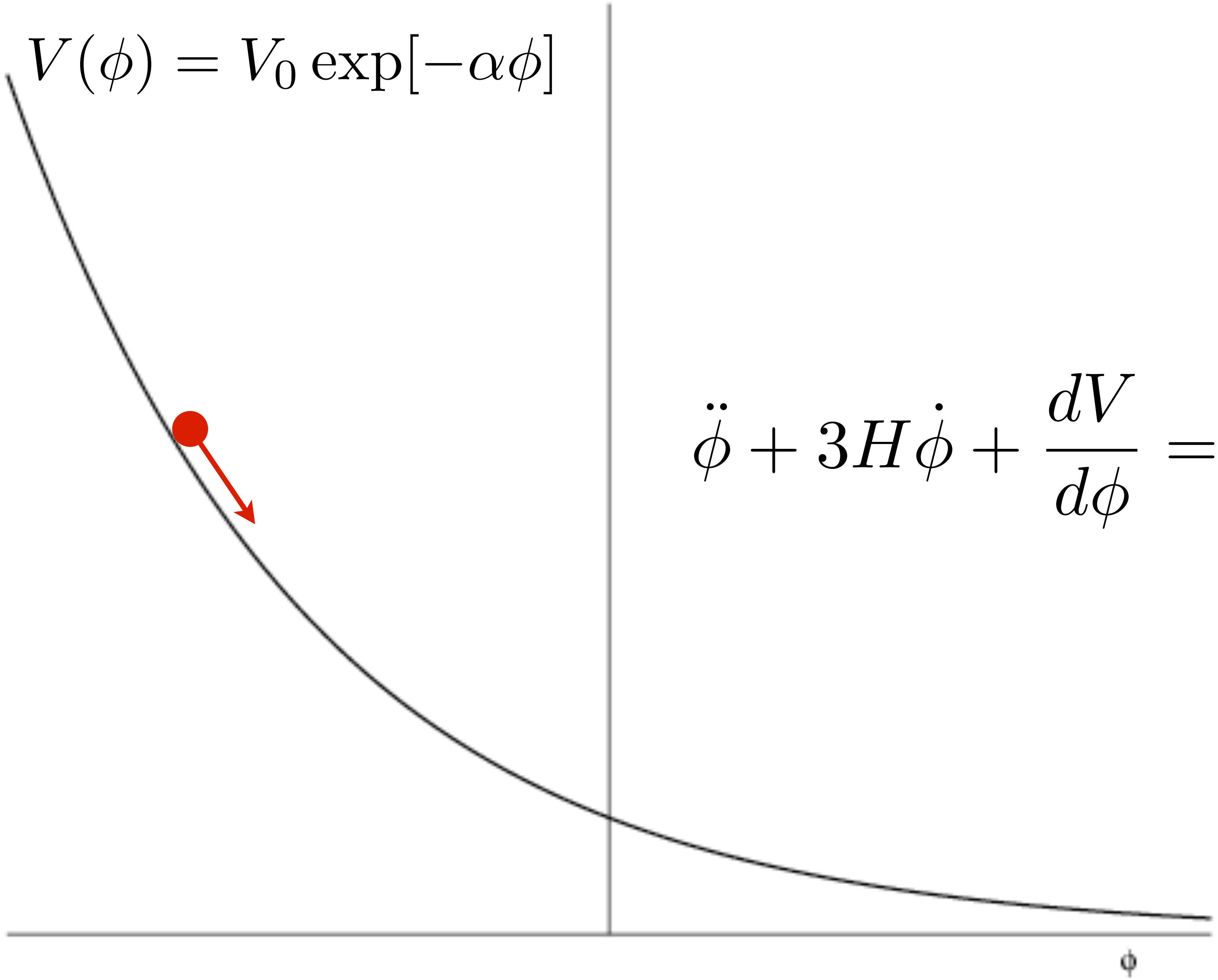

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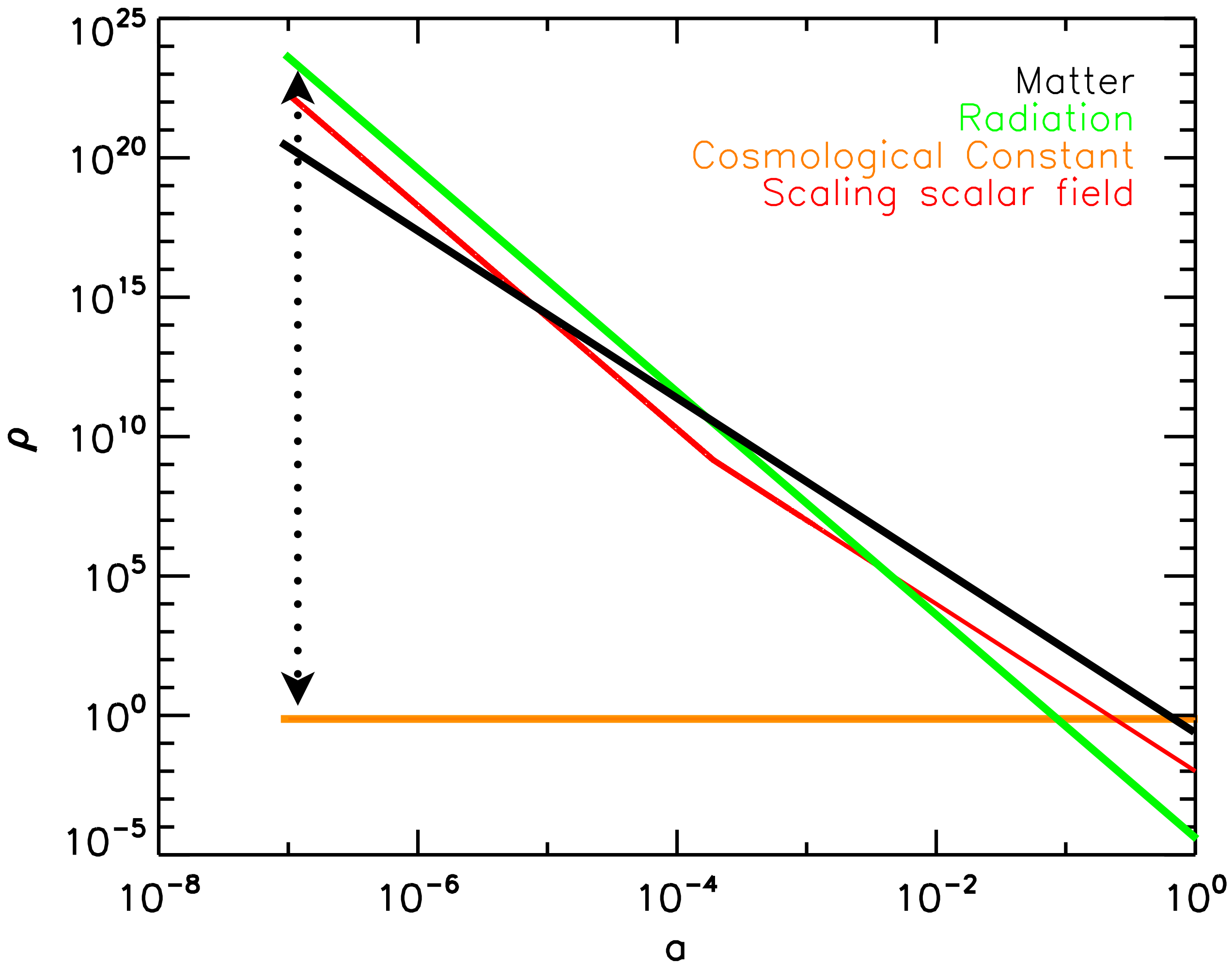

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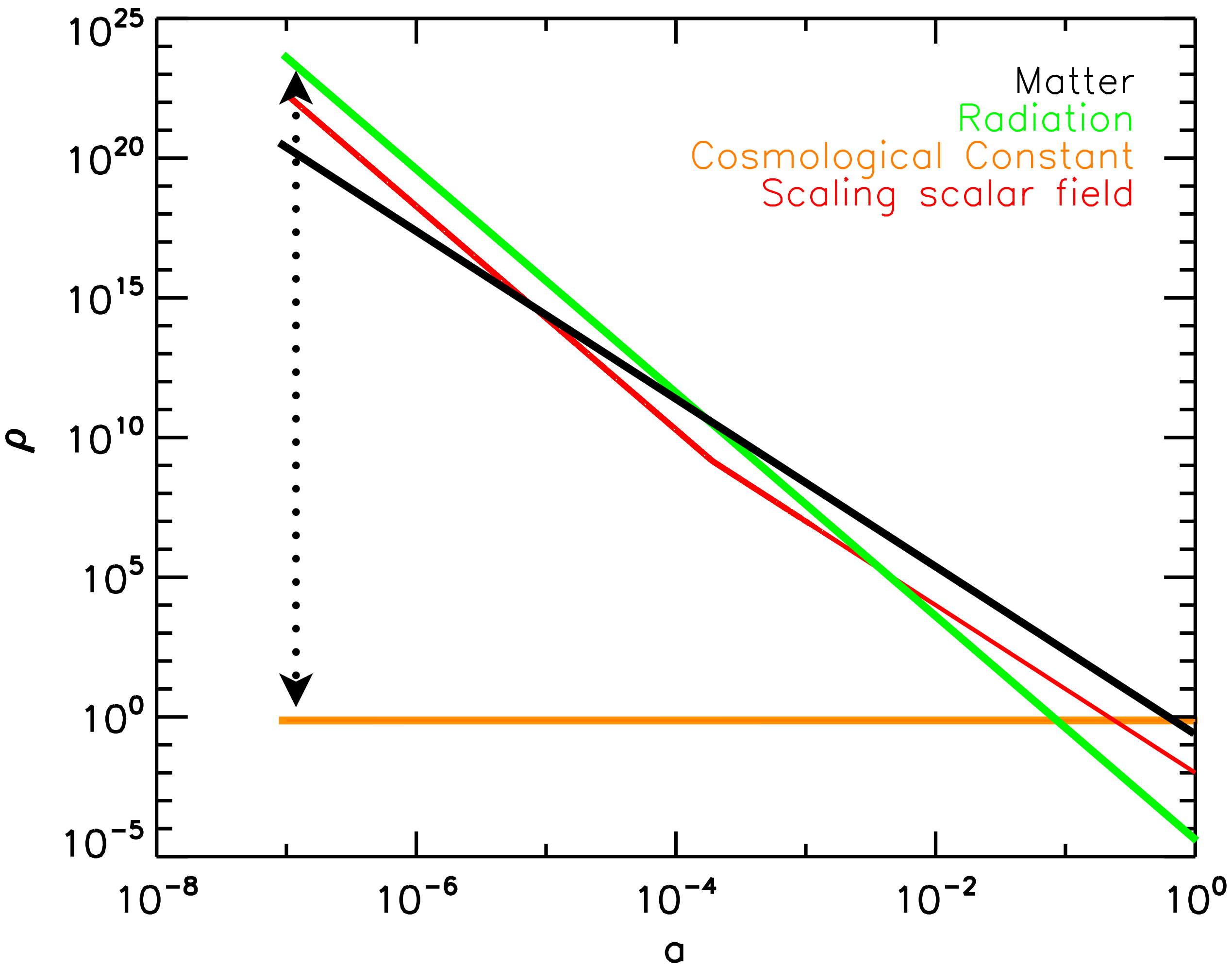

However, on the scaling solution

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Possible solutions to this issue amount to perturbing the potential (SUGRA), having a time-dependent slope $\alpha(t)$, or introducing an interaction of the field (coupled DE)

k-essence

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Generalised scalar field models whose Lagrangian density is given by a generic function $\mathcal{L}_\phi = p(\chi, \phi)$ of the scalar field ϕ and of its kinetic energy $\chi \equiv -(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ are called k-essence.

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This allows to consider models with negative kinetic energy (called phantom): $p(\chi, \phi) = -\chi - V(\phi)$

The energy, pressure, and equation of state for k-essence are:

$$\rho_\phi = 2\chi\partial p/\partial\chi - p \qquad p_\phi = p \qquad w_\phi = \frac{p}{2\chi\partial p/\partial\chi - p} \tag{75}$$

so that the condition for acceleration is given by:

$$|2\chi\partial p/\partial\chi| \ll |p| \tag{76}$$

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It is however possible to build Dark Energy models with a sound speed as low as matter $c_s^2 \approx 0$ so that Dark Energy density perturbations can survive also at sub-horizon scales.

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Interacting DE (Coupled and Extended Quintessence, Modified Gravity)	✔ a dynamical (scalar) degree of freedom	✔ fluctuations sourced by the interaction	✔ non-minimally coupled to matter

Interacting Dark Energy: coupled Quintessence (I)

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$$\nabla_\mu T^{\mu(\mathrm{DM})}_\nu + \nabla_\mu T^{\mu(\mathrm{b})}_\nu + \nabla_\mu T^{\mu(\mathrm{r})}_\nu + \nabla_\mu T^{\mu(\mathrm{DE})}_\nu = 0 \tag{80}$$

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So, if dark matter and a scalar field have opposite source terms

$$\nabla_\mu T^{\mu(\phi)}_\nu = -QT^{(\mathrm{DM})}\nabla_\nu\phi \qquad \nabla_\mu T^{\mu(\mathrm{DM})}_\nu = +QT^{(\mathrm{DM})}\nabla_\nu\phi \tag{82}$$

this corresponds to a direct interaction between the two fields

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$$\dot{\rho}_{\rm DM} + 3H\rho_{\rm DM} = +Q\rho_{\rm DM}\dot{\phi} \qquad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = -Q\rho_{\rm DM} \tag{83}$$

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The coupling term in the Klein-Gordon equation can be seen as a perturbation of the potential that results in a shallower effective potential for the scalar field.

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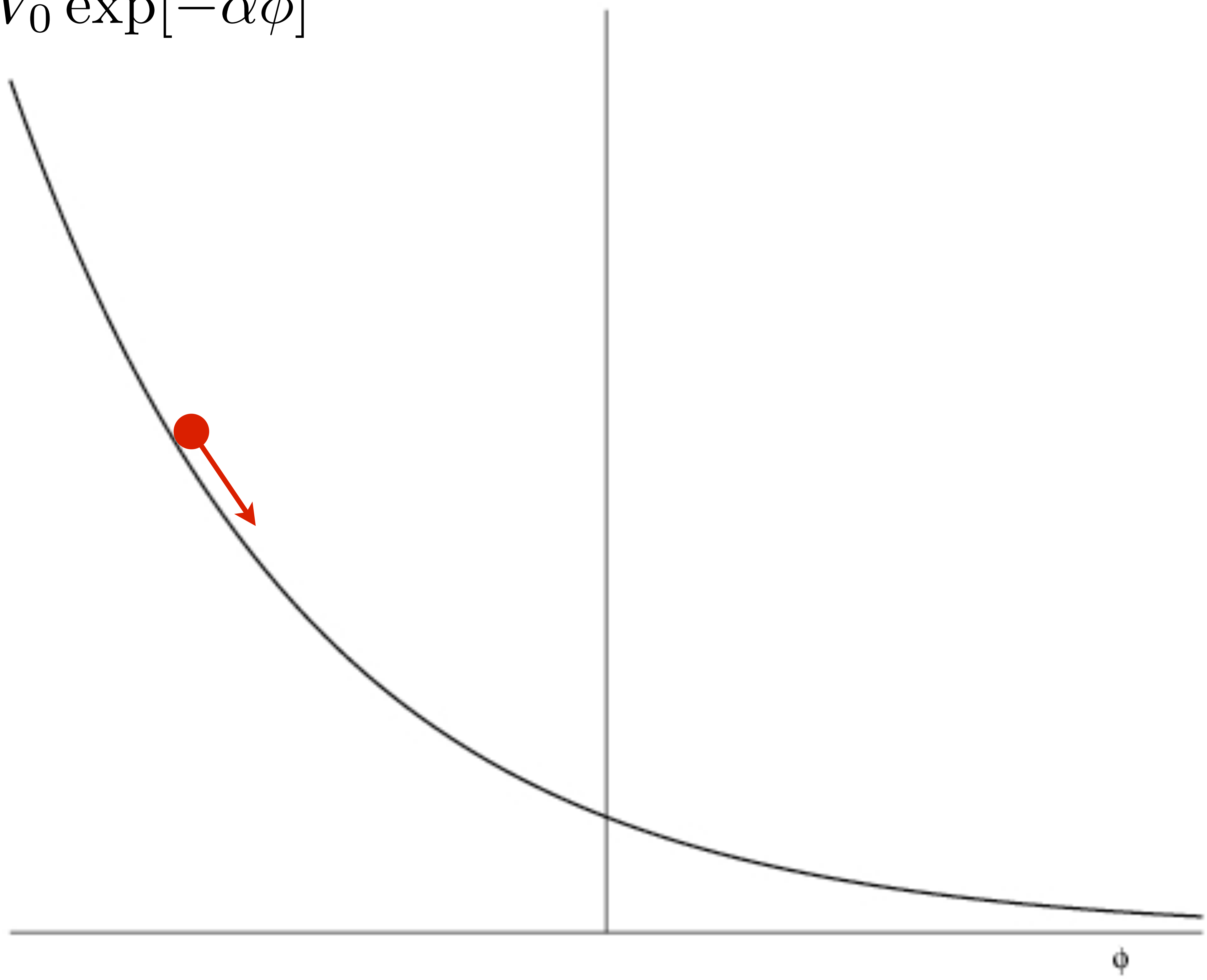

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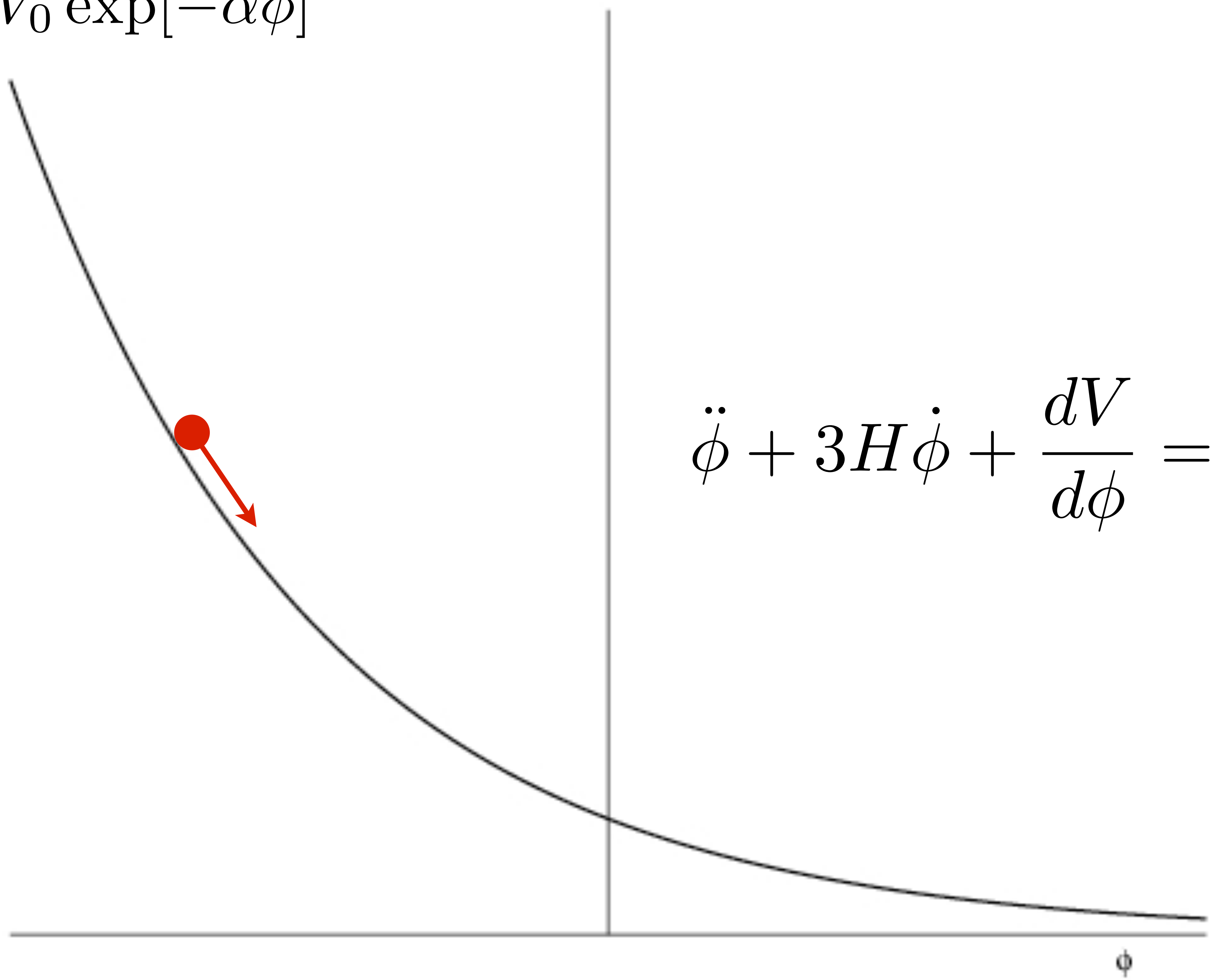

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$$+ \int Q \rho_{\rm DM} d\tilde{\phi}$$

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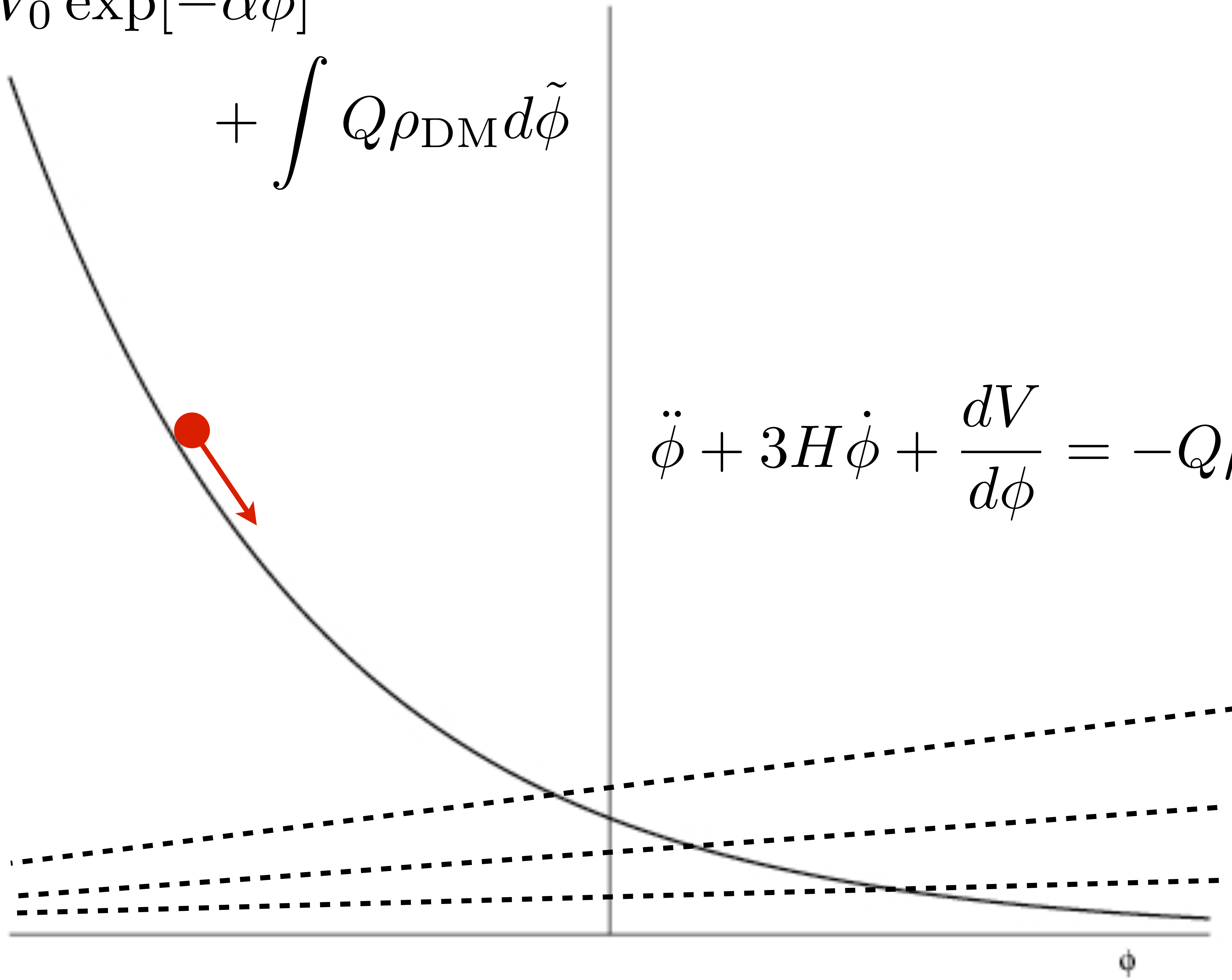

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The most significant property of coupled Quintessence is that the coupling determines a new type of scaling solution, called Φ-Matter Dominated Epoch (Φ-MDE), characterised by:

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The novel feature of this solution is that for $Q \ll 1$ and

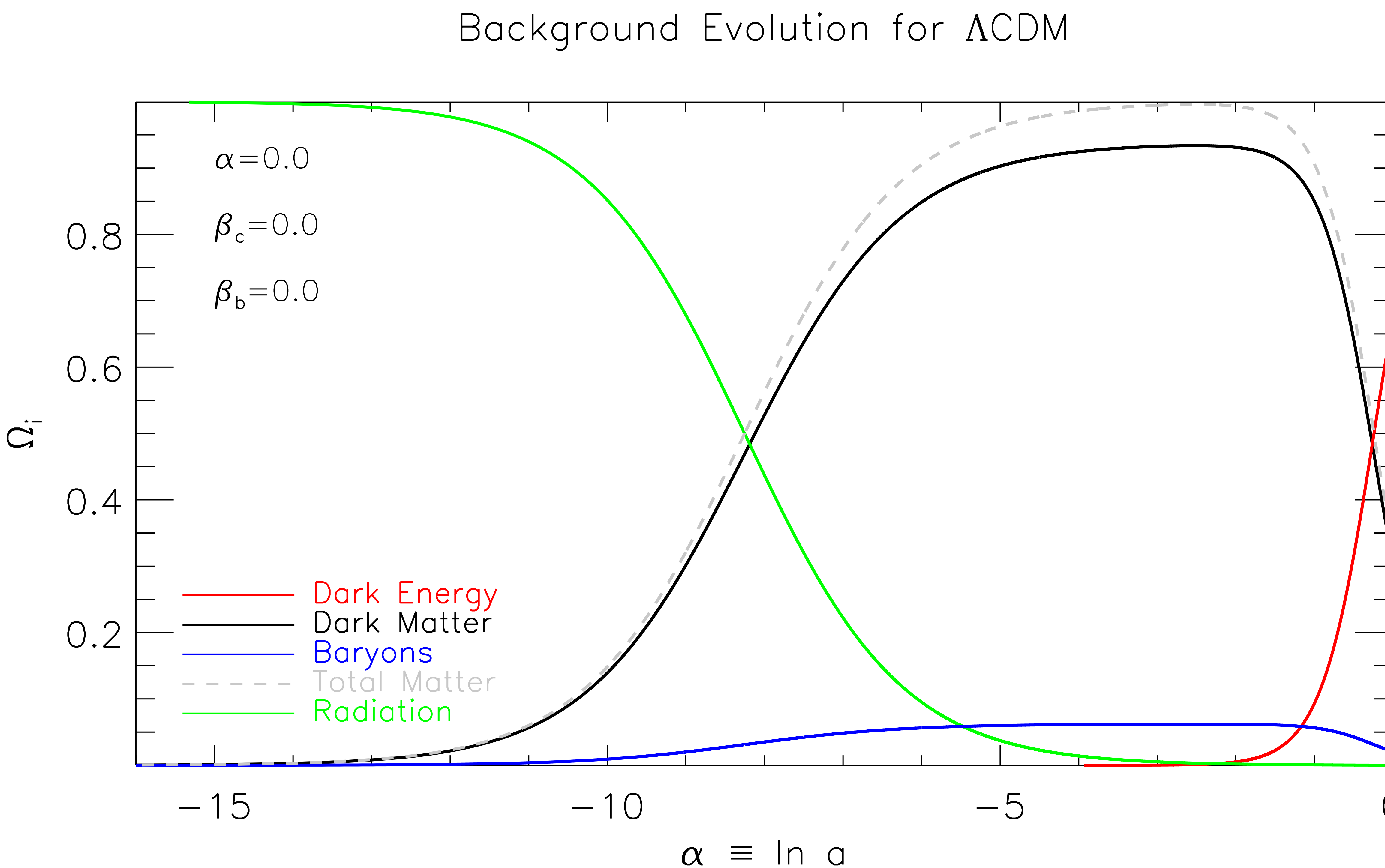
$$Q\left(Q + \frac{\alpha}{8\pi G}\right) > -\frac{3}{2} \tag{87}$$

it is a saddle point (metastable solution) that naturally evolves to an accelerated dark-energy dominated solution

Interacting Dark Energy: coupled Quintessence (V)

Consider $Q(\phi) \equiv -\sqrt{8\pi G}\beta$

The interaction modifies the cosmic background evolution



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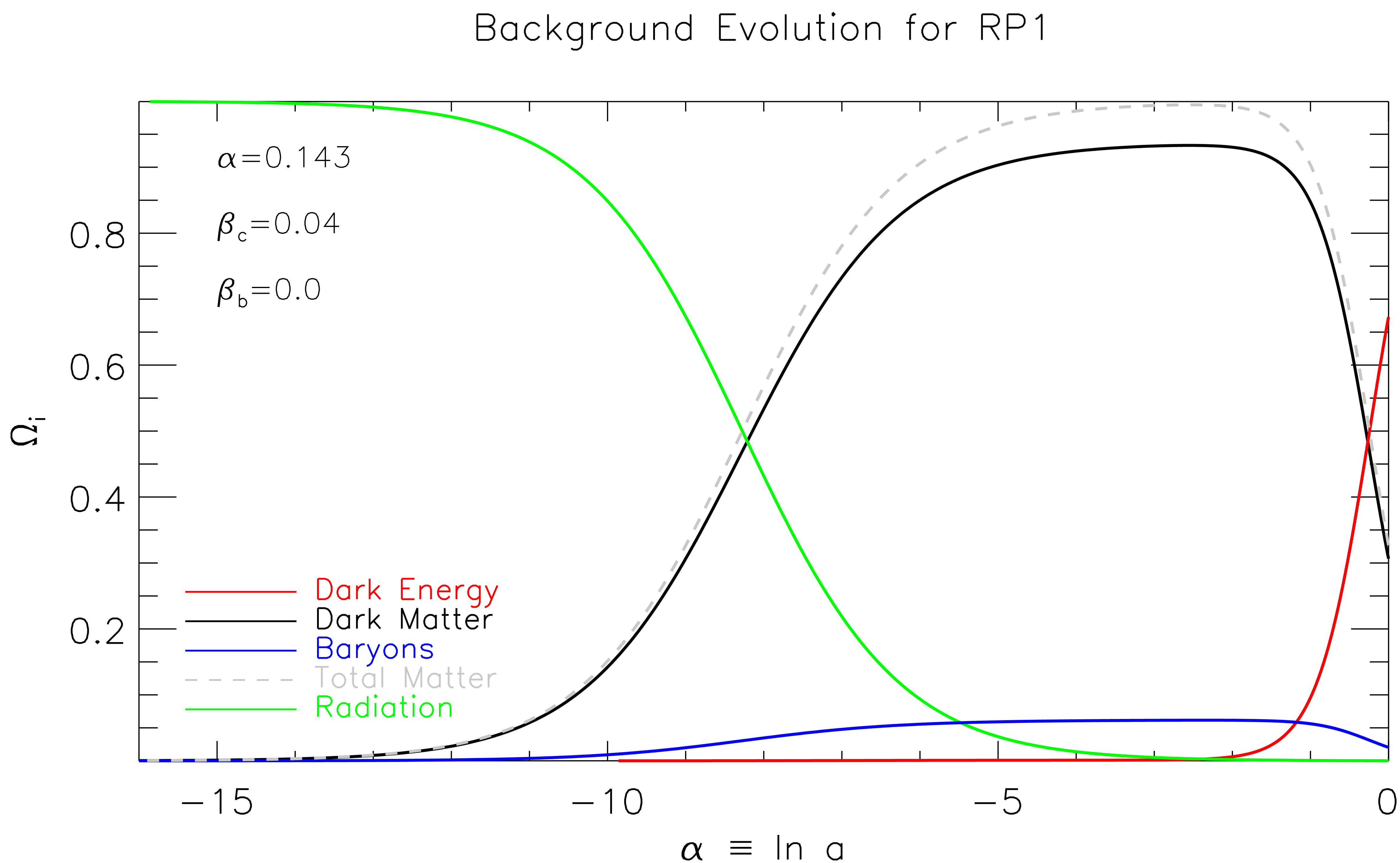

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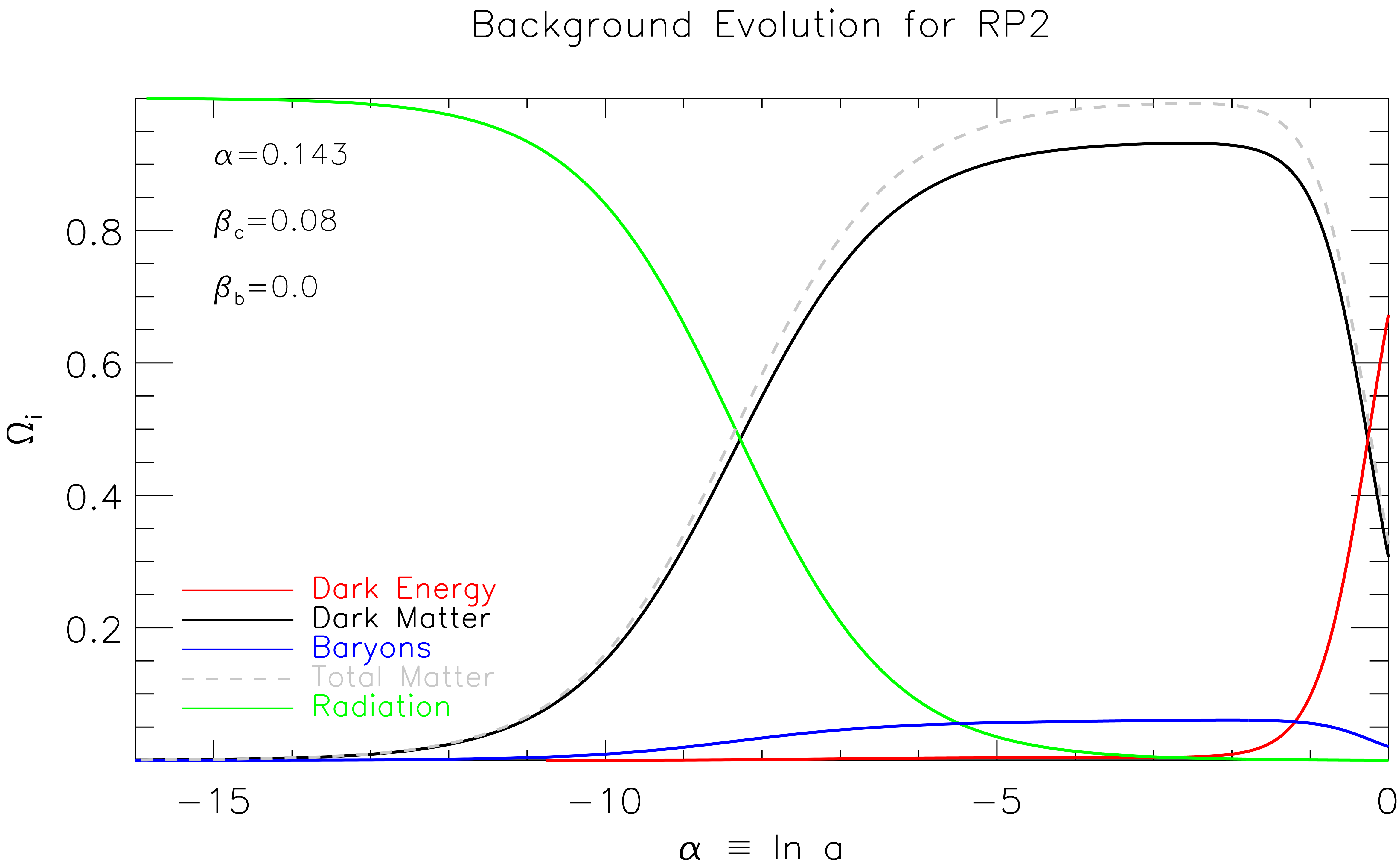

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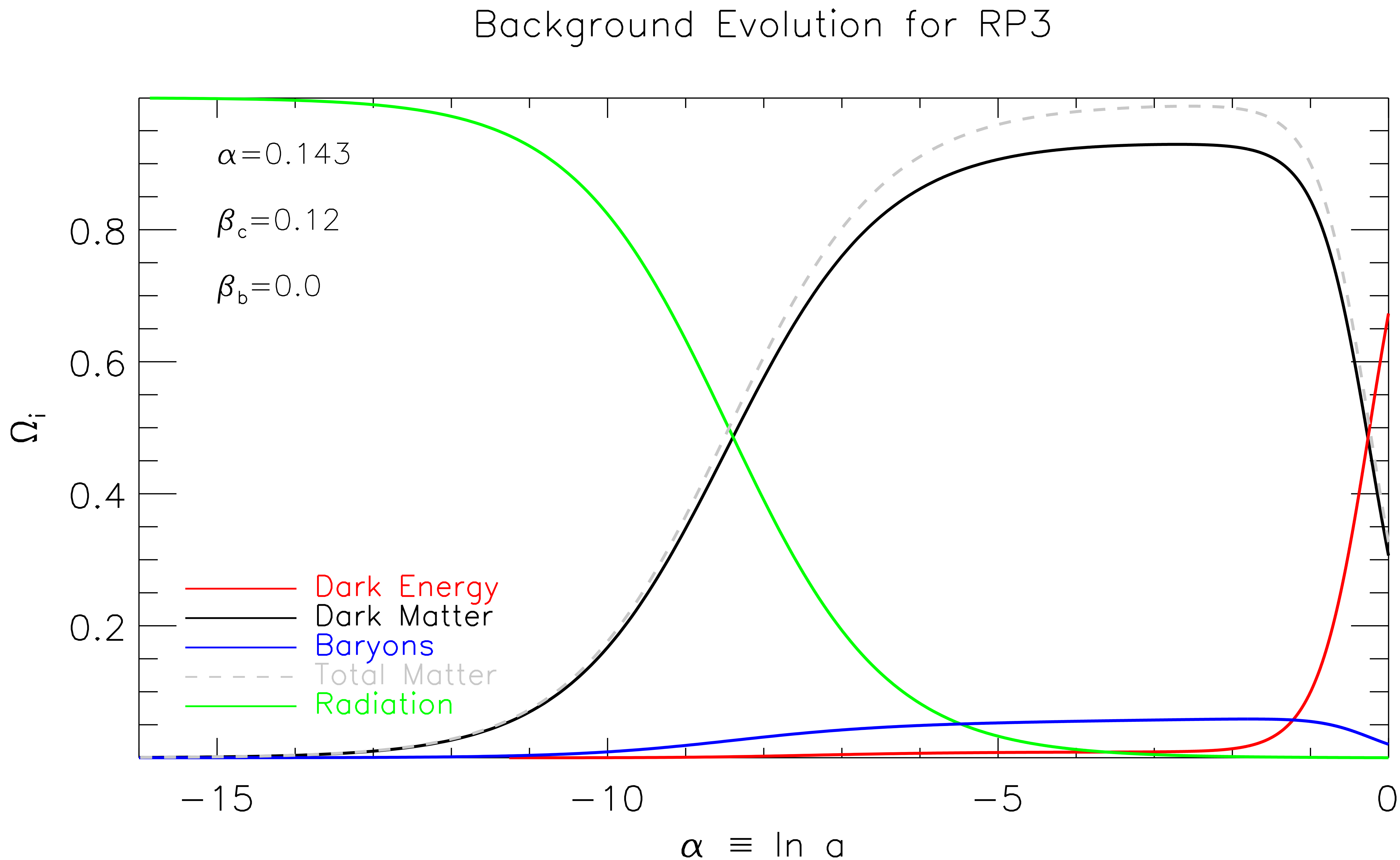

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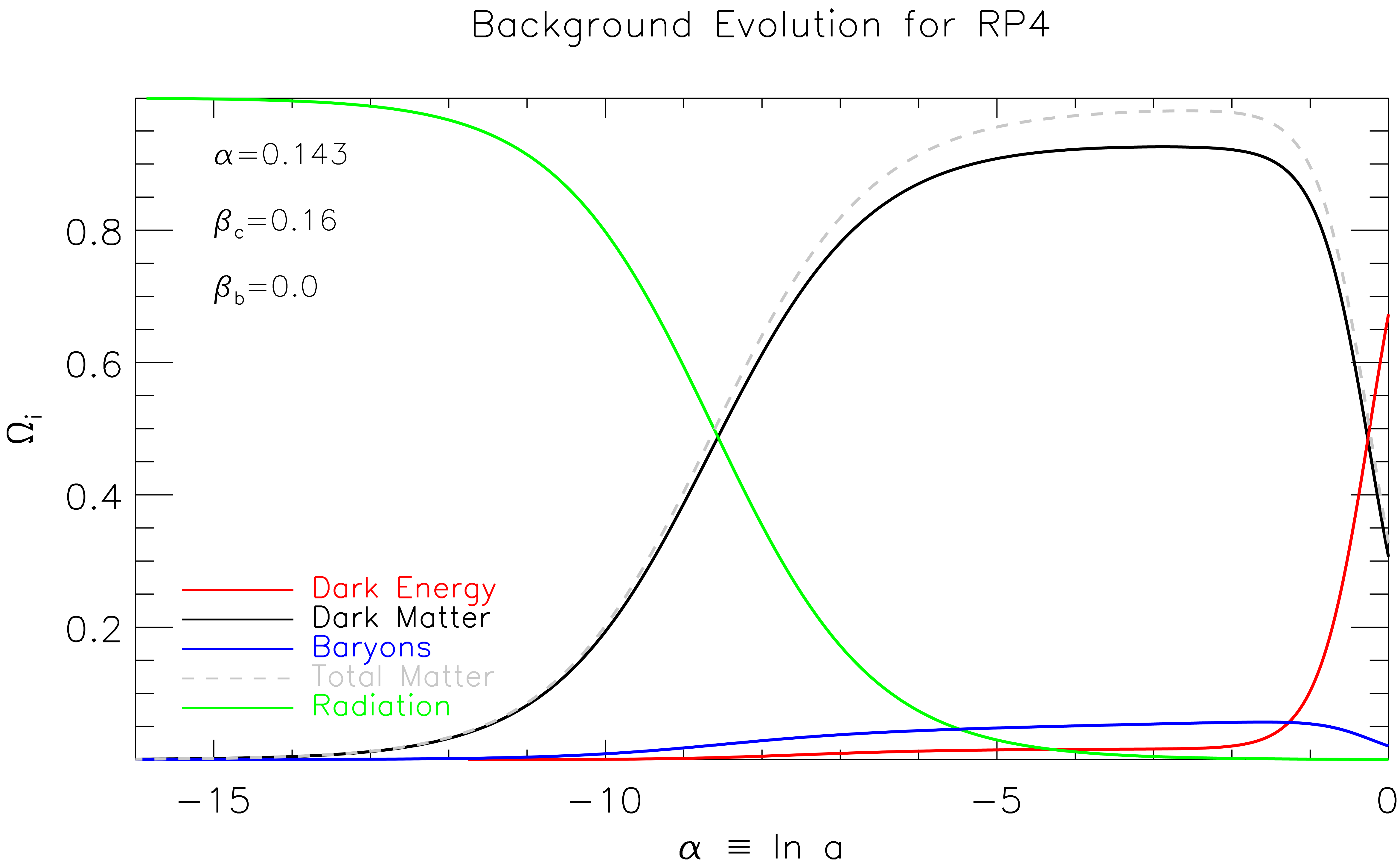

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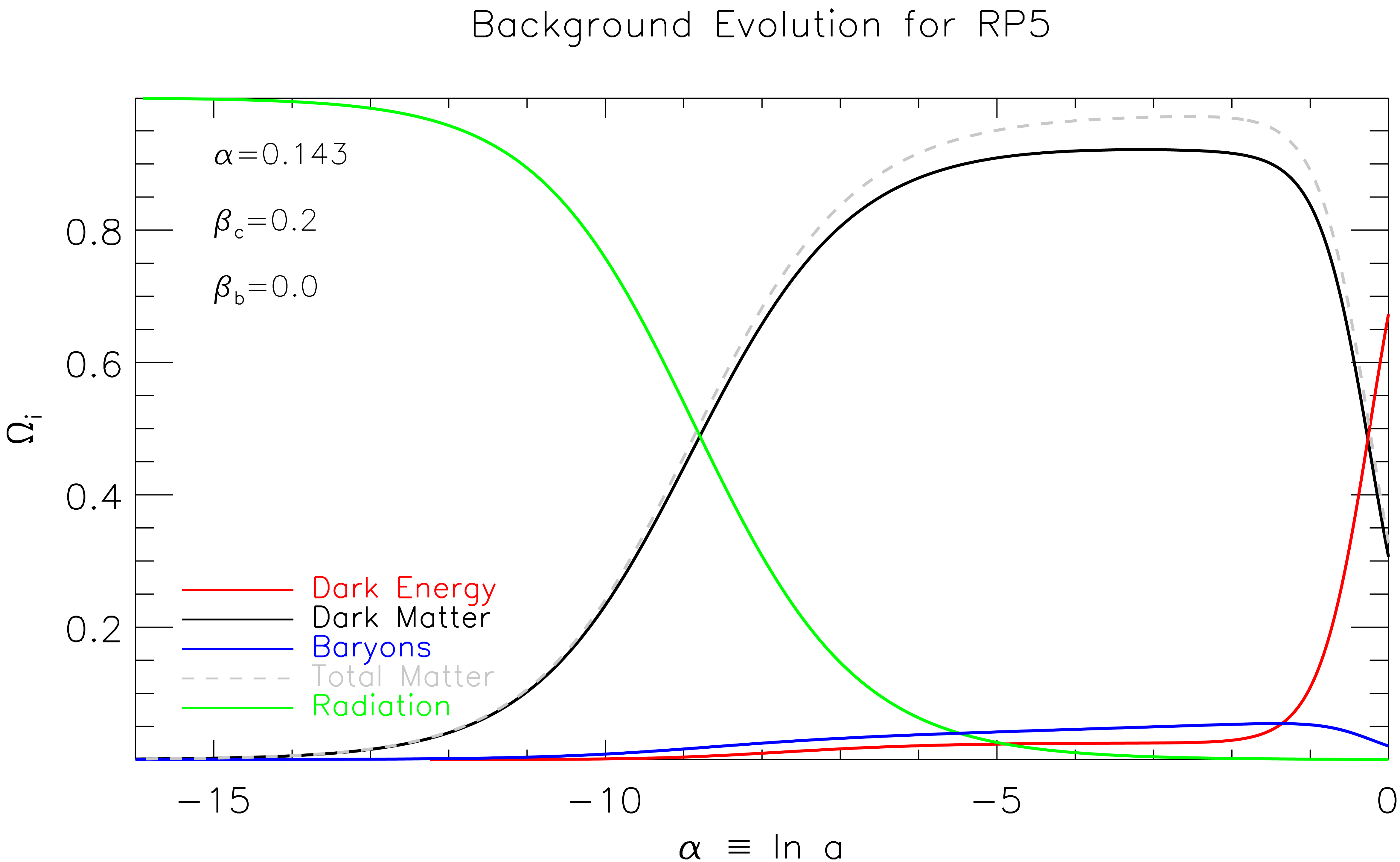

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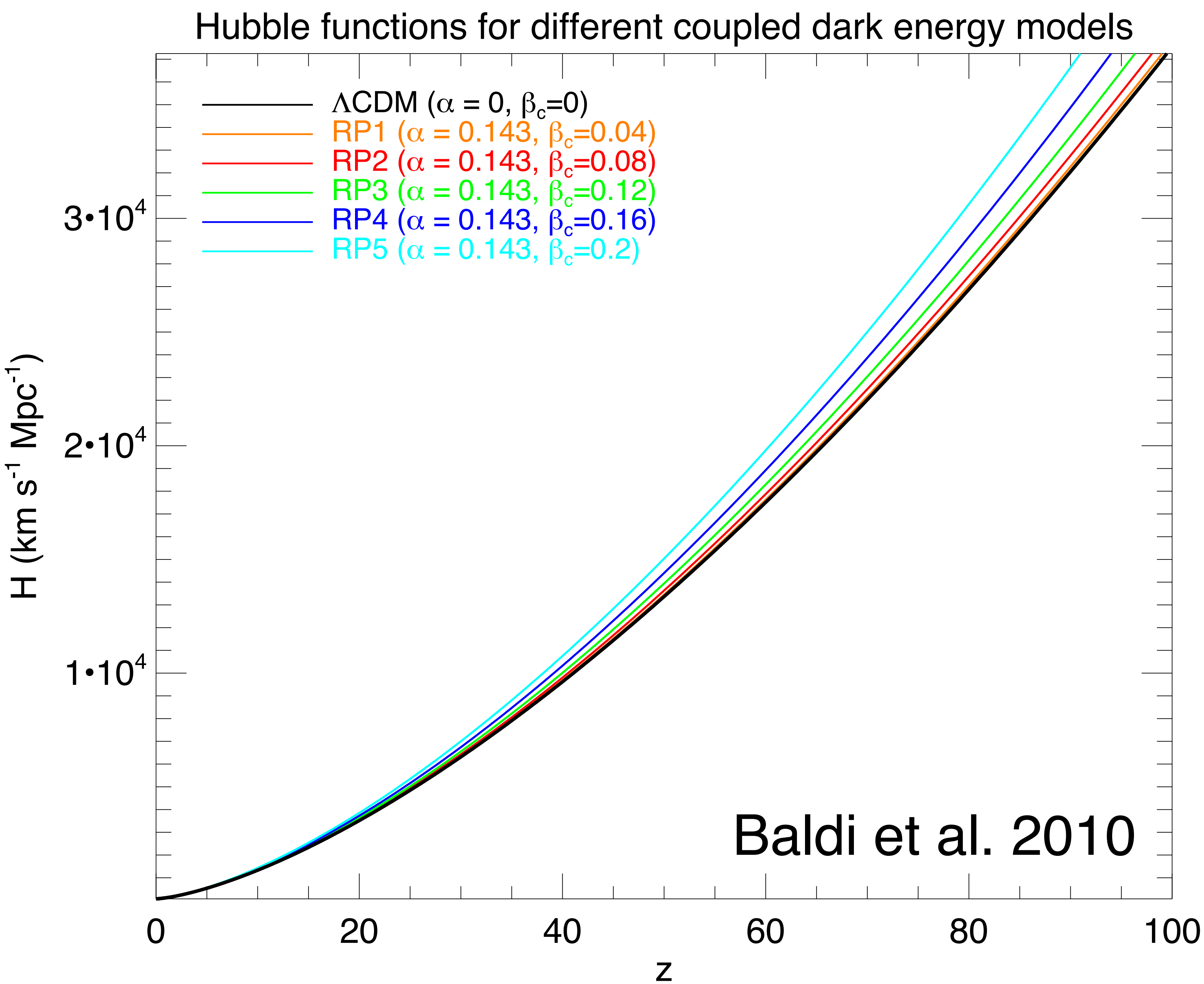

Interacting Dark Energy: coupled Quintessence (VII)

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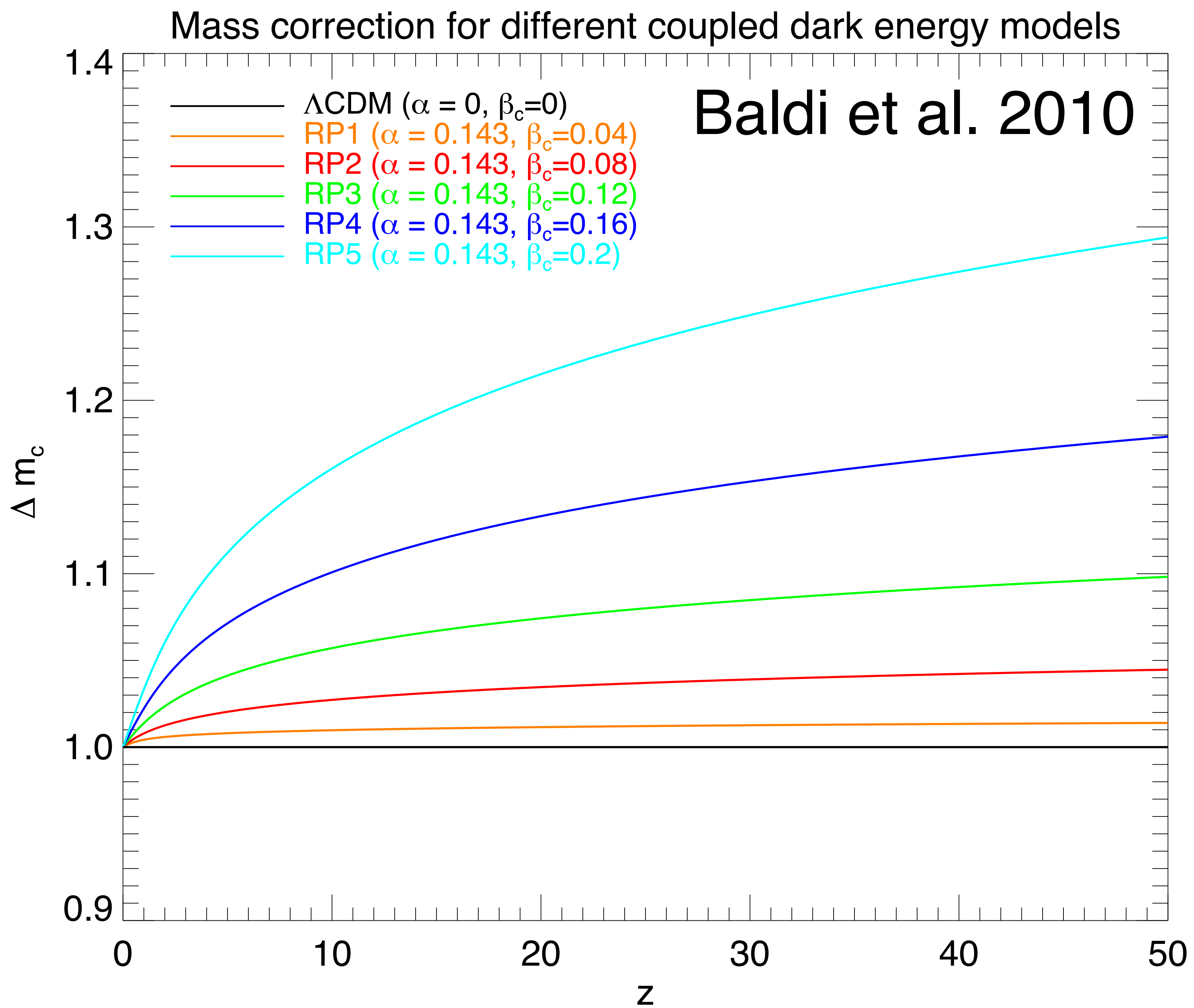
The mass variation, along the modified background evolution, corresponds to a transfer of energy from the DM field to the scalar field



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$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu}, \Psi_m) \qquad (88)$$

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$$f_R R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + g_{\mu\nu}\Box f_R = 8\pi G T_{\mu\nu} \qquad (89)$$

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trace →

$$3\Box f_R + f_R R - 2f(R) = -8\pi G(\rho - 3p) \tag{90}$$

where we have defined $f_R \equiv df/dR$ (91)

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A particularly relevant model of *f(R)* gravity is given by the choice (Hu & Sawicki 2007):

$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1} \qquad m^2 \equiv \frac{8\pi G \rho_0}{3} \tag{92}$$

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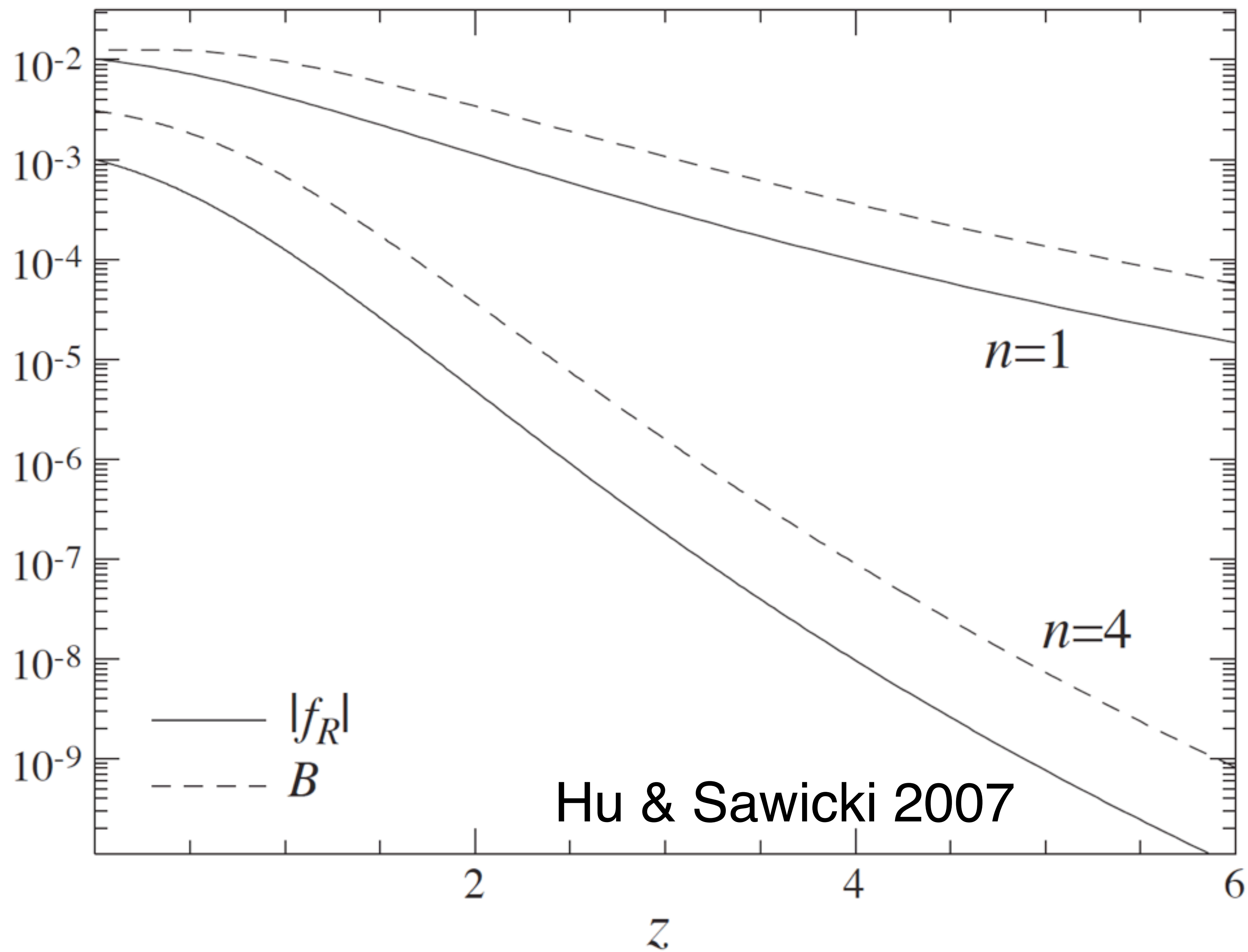
In this setup the *f(R)* model will differ from the standard ΛCDM cosmology only at the level of linear and non-linear perturbations

Modified Gravity: *f(R)* (III)

$$f_R \to 0 \Rightarrow f(R) \to \mathrm{GR}$$



Recap Lecture 2

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and by perturbing the EM tensor of a perfect fluid

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one can then obtain the equations for the evolution of density and velocity perturbations

$$\delta' + 3\mathcal{H}(c_s^2 - w)\delta = -(1+w)(\theta + 3\Phi')$$

$$\theta' + \left[\mathcal{H}(1-3w) + \frac{w'}{1+w}\right]\theta = -\nabla^2\left(\frac{c_s^2}{1+w}\delta + \Psi\right)$$

that describe how perturbations grow through grav. instability

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The source term is given by the Poisson equation

$$\nabla^2\Phi = 4\pi G a^2 \rho\delta = \frac{3}{2}\frac{8\pi G\rho}{3H^2}a^2H^2\delta = \frac{3}{2}\Omega\mathcal{H}^2\delta$$

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or scalar field models (Quintessence, k-essence)

$$\mathcal{L}_\phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \qquad \mathcal{L}_\phi = p(\chi,\phi)$$

Recap Lecture 2

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Scalar field models can also have interaction terms with matter

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Another alternative is to directly modify the gravitational Action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_m(g_{\mu\nu}, \Psi_m)$$

which can give a similar background expansion as ΛCDM but a different growth of perturbations