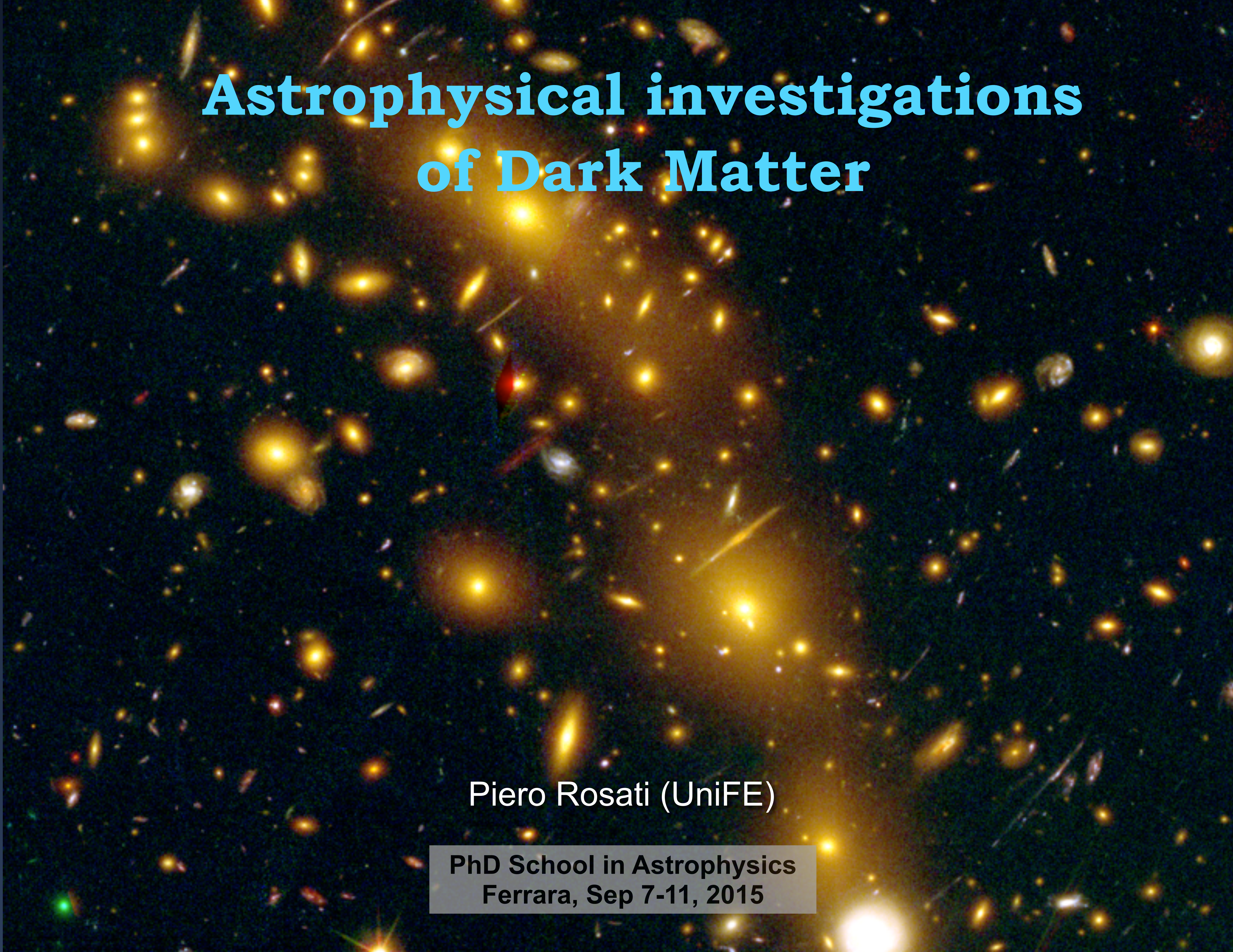
# Astrophysical investigations of Dark Matter



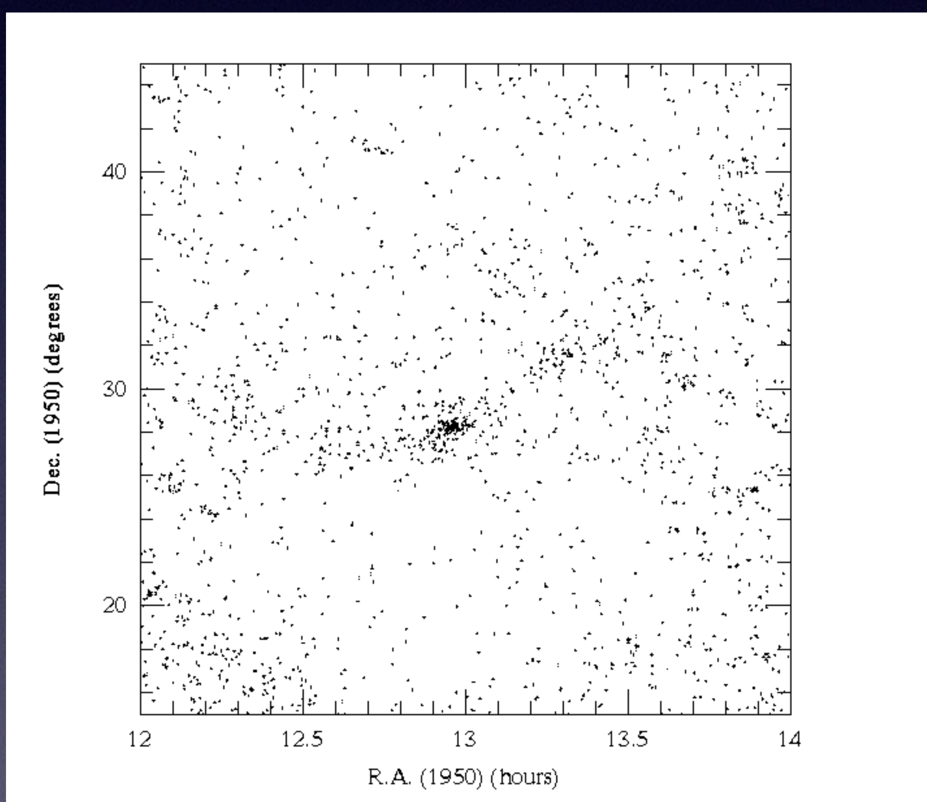
Piero Rosati (UniFE)

PhD School in Astrophysics
Ferrara, Sep 7-11, 2015

# Discovery of Dark Matter

Dynamics of cluster galaxies

Fritz Zwicky (1898-1974)





Zwicky applied the viral theorem to Coma:

2T+U=0 ➜ $$\frac{GM}{R} \approx \sigma_v^2$$

..and finds that $M_{cluster} > 10 \sum_i M_{gal}$

# Discovery of Dark Matter

Fritz Zwicky (1898-1974)

**Die Rotverschiebung von extragalaktischen Nebeln**

von **F. Zwicky.**

(*Helv. Phys. Acta*, 6, No. 2, p. 110, 1933)

von Beobachtungen an leuchtender Materie abgeleitete[1]). Falls sich dies bewahrheiten sollte, würde sich also das überraschende Resultat ergeben, dass dunkle Materie in sehr viel grösserer Dichte vorhanden ist als leuchtende Materie.

↓

Should this be confirmed, we would get the surprising result that **dark matter** is present in much greater amount than luminous matter.

Then published on ApJ in 1937

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 86 OCTOBER 1937 NUMBER 3

ON THE MASSES OF NEBULAE AND OF CLUSTERS OF NEBULAE

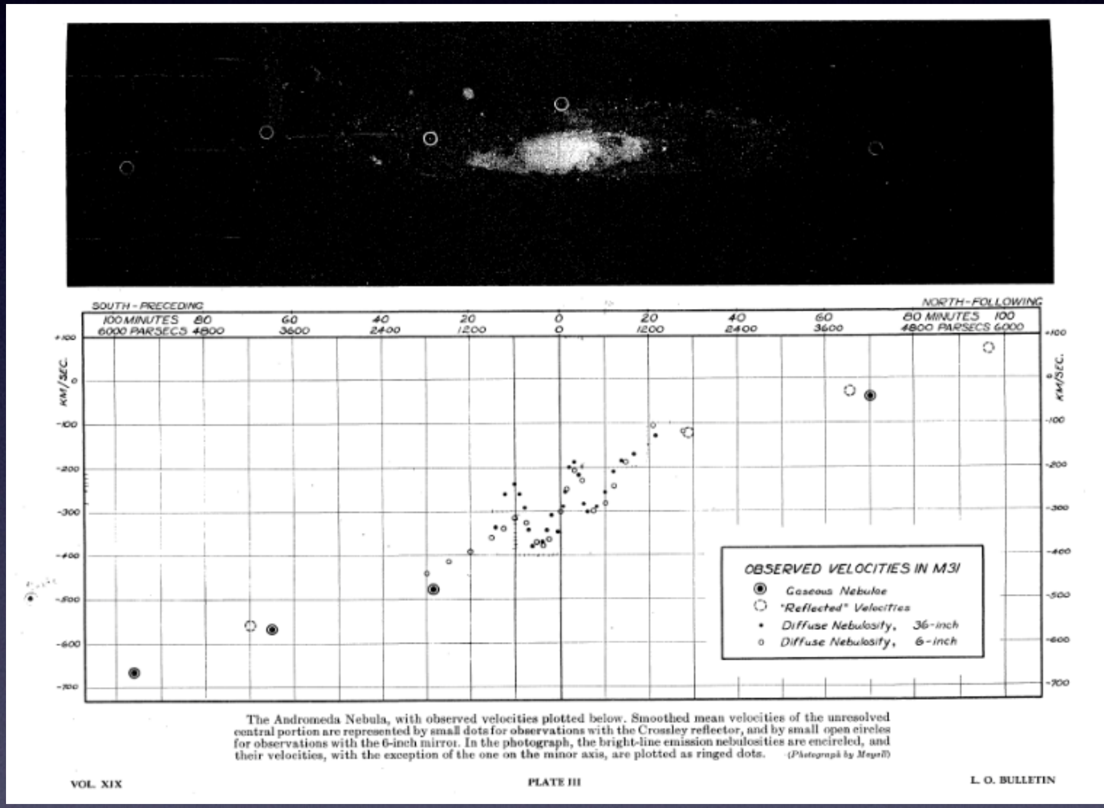
- Sinclair Smith (ApJ, 1936) also noted that in Virgo there was a mass mismatch:
  "*It is possible that both figures [cluster mass and lum. mass] are correct and and that the difference represents a great mass of intra-nebular material in the cluster*"

[6] F. Zwicky has pointed out (*Helv. Phys. Acta*, 6, No. 2, p. 110, 1933) that the velocity range in the Coma Cluster indicates non-luminous matter which is some four hundred times the amount of the observed luminous material.

# Discovery of Dark Matter

- Note that before Zwicky, Jacobus Kapteyn (Apj, 1922: "*First Attempt at a Theory of the Arrangement and Motion of the Sidereal System*", had first used the term of dark matter (possibility of using stellar dynamics to weigh luminous+non-luminous matter)

- 1939: Babcock notes that M31 rotation curve remains flat at large radii: "*The obvious interpretation of the nearly constant velocity for 30' outward is that a that a very great portion of the mass of the nebula must lie in the outer regions*"



The Andromeda Nebula, with observed velocities plotted below. Smoothed mean velocities of the unresolved central portion are represented by small dots for observations with the Crossley reflector, and by small open circles for observations with the 6-inch mirror. In the photograph, the bright-line emission nebulosities are encircled, and their velocities, with the exception of the one on the minor axis, are plotted as ringed dots. (Photograph by Mayall)

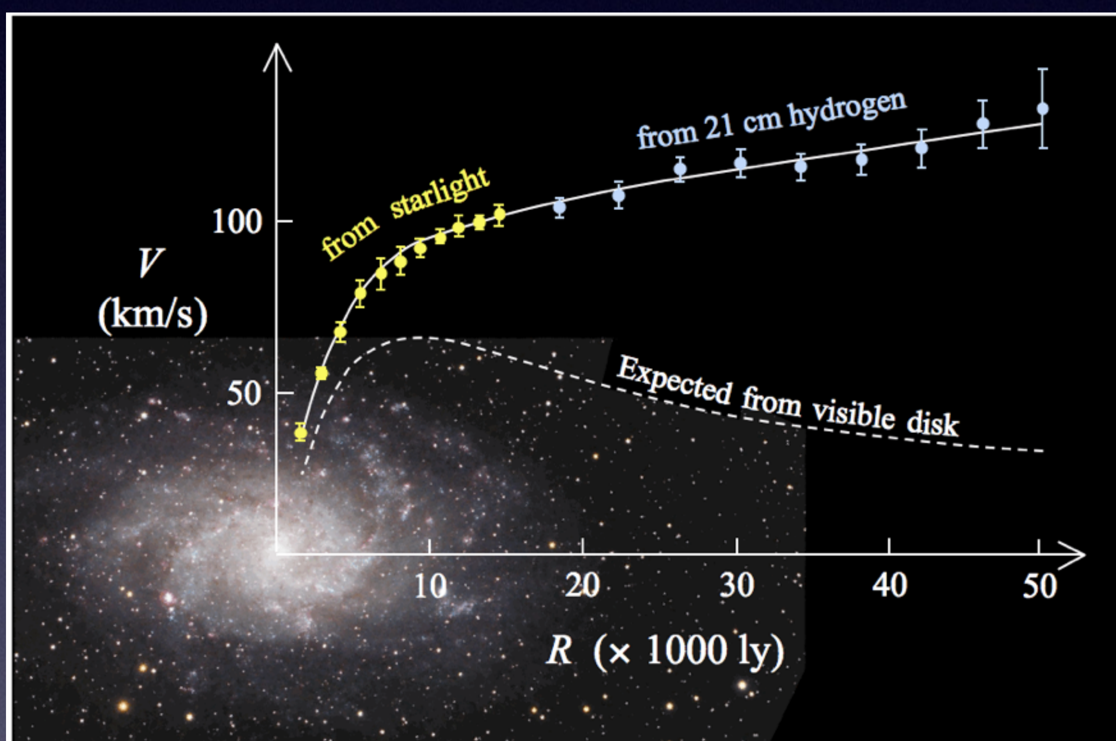
- 1959: Kahn and Woltjer on Local Group scale: "*Local Group galaxies [M31, MW] can be dynamically stable only if it contains an appreciable amount of intergalactic matter… The Discrepancy seems to be well outside the observational errors*"

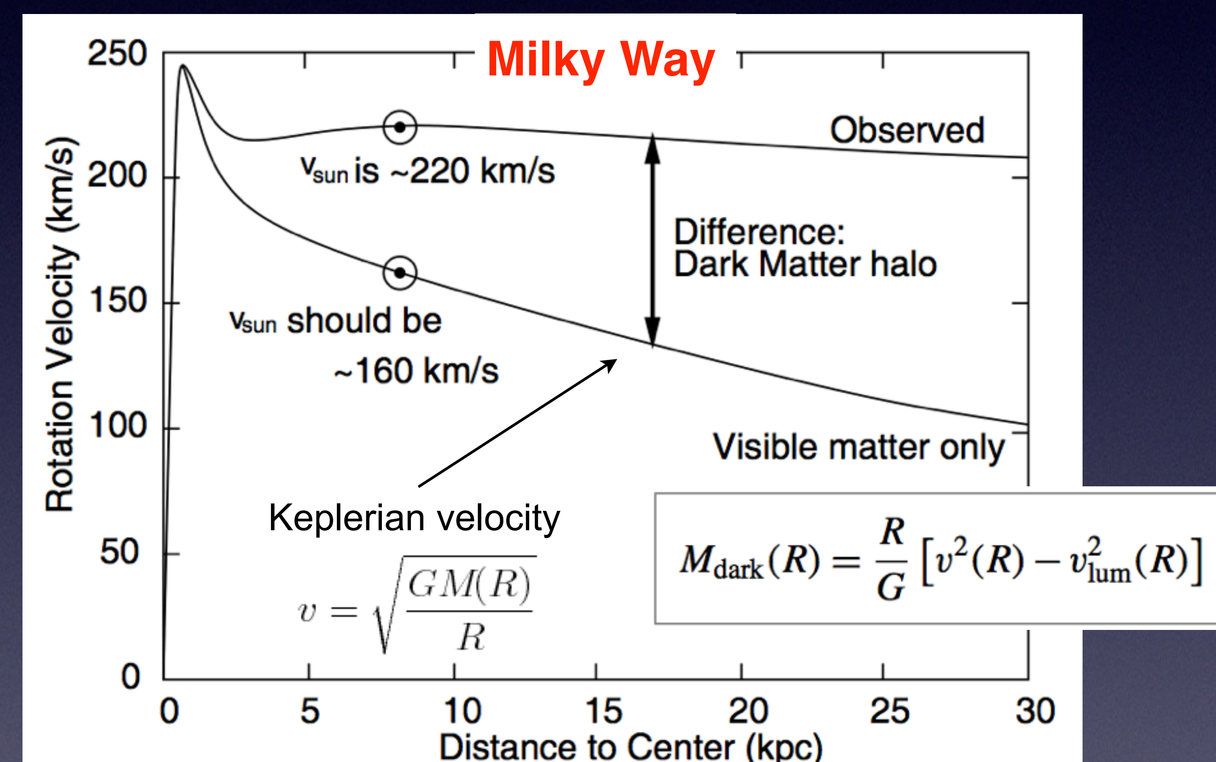
- *However, this didn't seem to be a big deal until the late seventies (Zwicky's obituary doesn't even mention DM..)*

# Discovery of Dark Matter

Galaxy rotation curves

- Rotationally supported systems (spiral galaxies): rotation curves (from the 70-80s): Lubin, Roberts, Bosma, Freeman, et al.




- From the 80s:
  DM becomes a key component on cosmological scale (Peebles and many others)

# Rotation curves in disk galaxies (exercise)

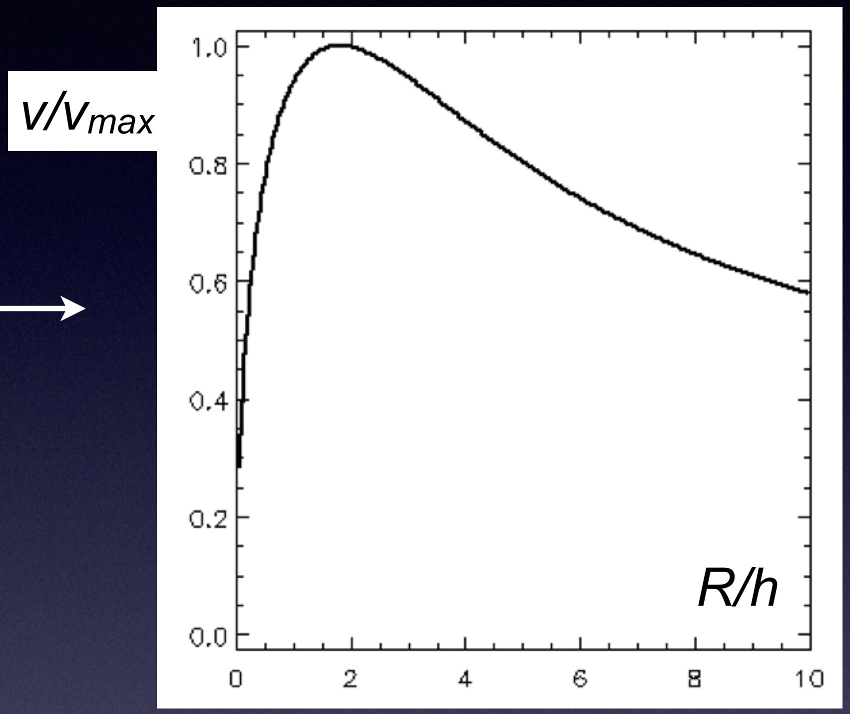
Disk galaxies have an exponential surface brightness profile: $I(R) = I_0 \exp(-R/h)$

The mass of the luminous mass can be measured assuming a constant M/L ratio with radius. Then:

$$M_{lum}(R) \propto L(R) = \int_0^R 2\pi r I_0 e^{-r/h}$$

Prove that the circular velocity of the stars in the disk is:

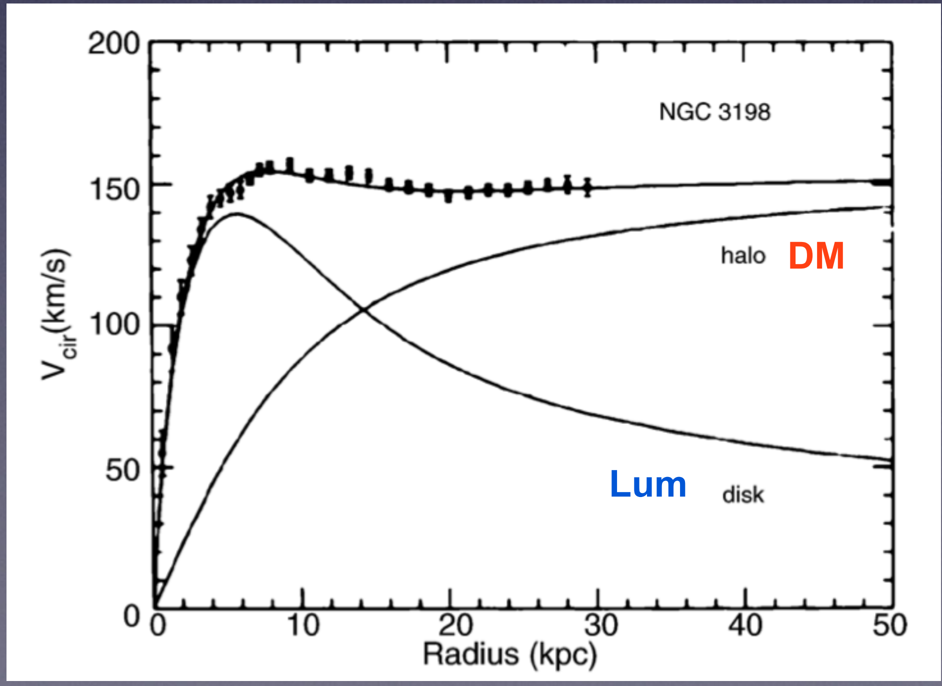
$$v^2 = \frac{GM_{lum}(R)}{R} = \ldots (x = R/h) \ldots \propto h(1/x - e^{-x}/x - e^{-x}) \longmapsto$$




Note: $v_{max} \sim h^{1/2} \rightarrow L \sim h^2 \sim v^4_{max} \rightarrow$ Tully-Fisher relation

**Dark matter**:

- Dynamically dominant at large radii (fraction of 50% Sa/Sb galaxies, up to 90% in dwarf galaxies)
- Distribution more extended than gas and stars (up 50-100 kpc)
- Distribution can be studied with
  - kinematics (stars, gas, 21 cm HI, satellites)
  - gravitational lensing
  - X-ray observations of early type galaxies



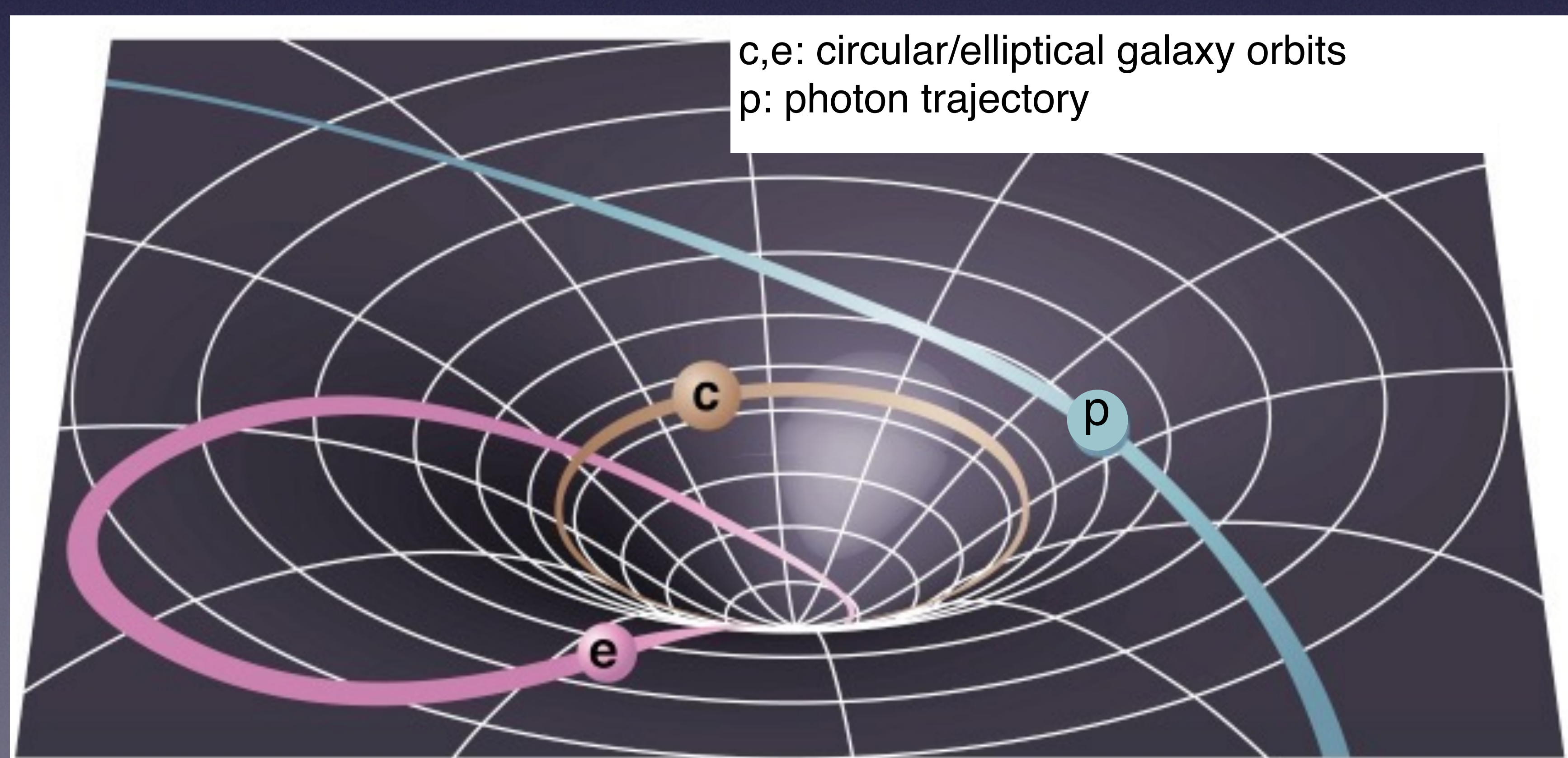
# How to measure total (DM) mass
# (from galaxies to clusters and the entire Universe)

There are essentially (only) two ways…

- Use the motion of matter (test particles: stars, galaxies, gas)
- Use the motion of light (gravitational lensing)

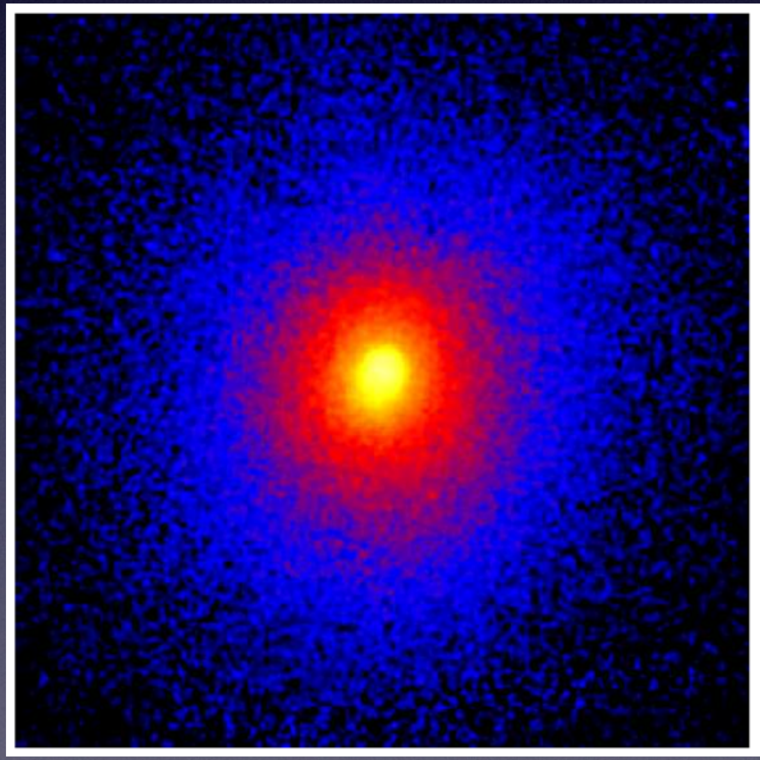
➔ to probe space-time curvature
mass distribution

GR

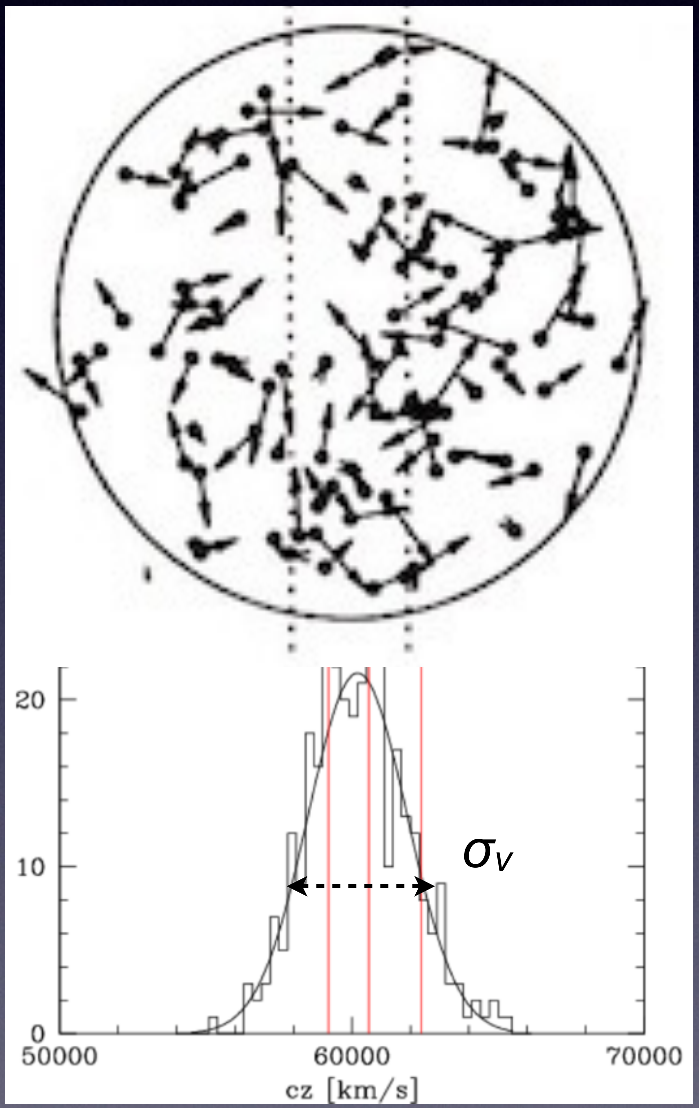


# How to measure total (DM) mass
# (from galaxies to clusters and the entire Universe)

## For galaxy clusters



X-ray hydrostatic equilibrium
(test particles: gas particles)

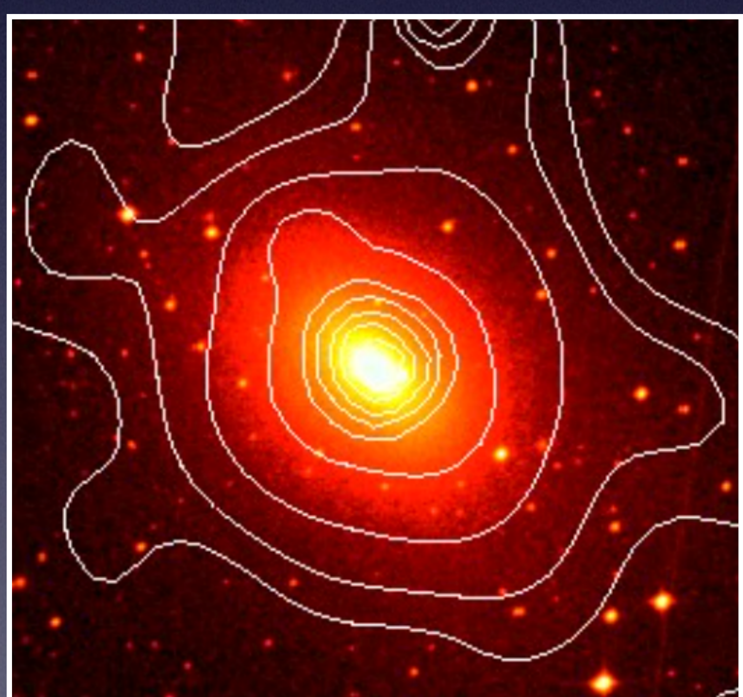



Galaxy dynamics
(test particles: galaxies)

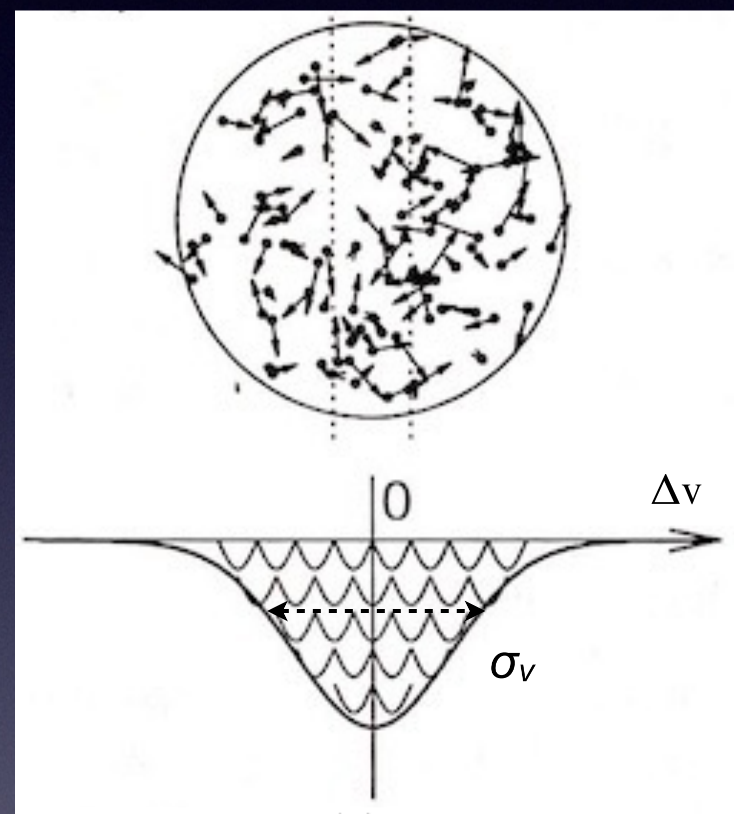
Gravitational lensing
(test particles: photons)

# How to measure total (DM) mass
# (from galaxies to clusters and the entire Universe)

## For spheroidal galaxies (pressure supported)



X-ray
hydrostatic equilibrium
(not as easy as in clusters)




$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{GM(r)}{r^2},$$

with $\quad p = \rho\sigma_v^2$

$$\rho(r) = \frac{\sigma_v^2}{2\pi G}\frac{1}{r^2}$$

Internal stellar
velocity dispersion
(from abs lines)

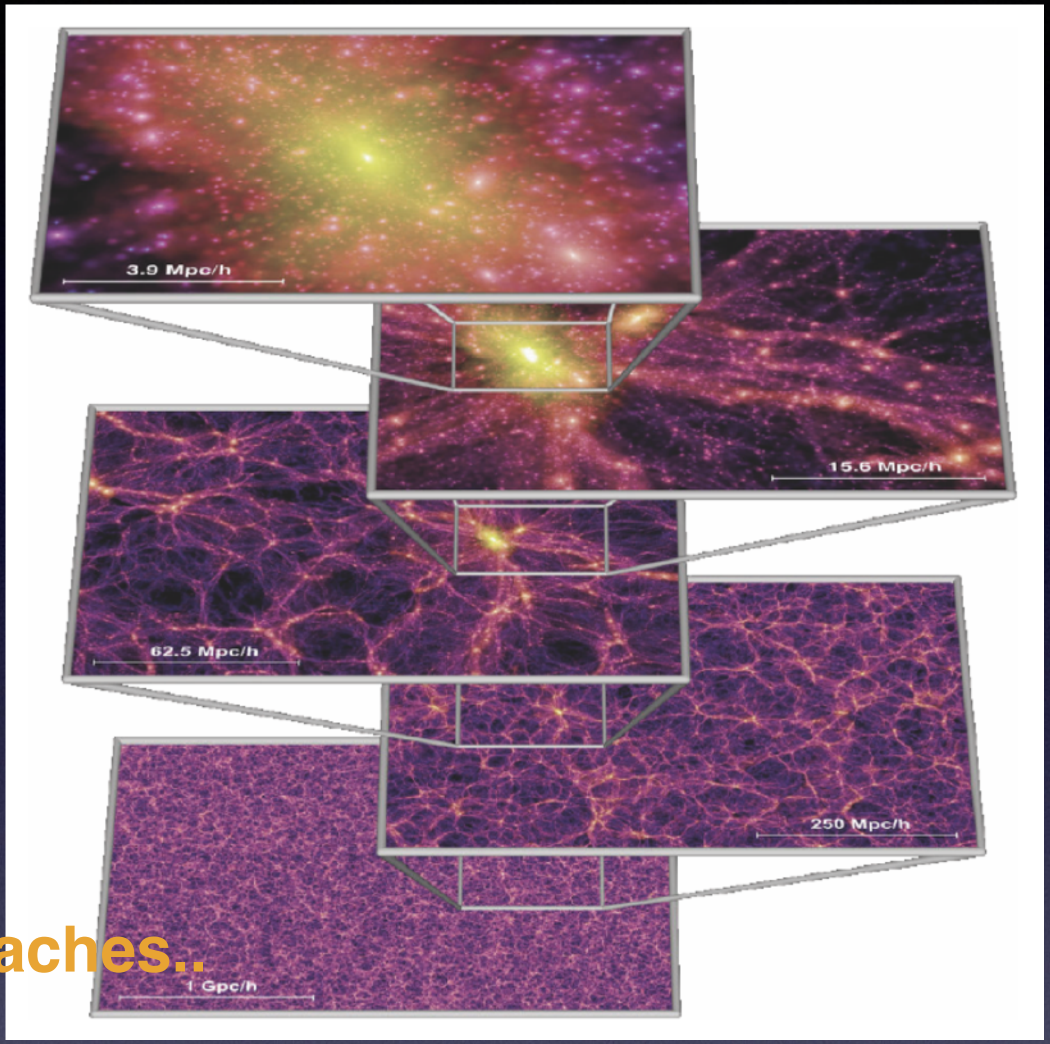
Gravitational lensing

# Total mass-energy density census at varying scales

## How can we study the structure of the Universe ?

- We can probe it with observations at three different levels of density perturbations…

   1. universal background effects: ($\rho$=$\rho_{\text{backgr}}$)
   (age, distances)
   2. linear perturbations ($\delta\rho/\rho \ll 1$)
   (clustering on large scales, CMB)
   3. non-linear perturbations ($\delta\rho/\rho \gtrsim 1$)
   (formation of collapsed objects,
   halo mass density profiles, structure of halos)




- **On cosmological scale, two complementary approaches..**

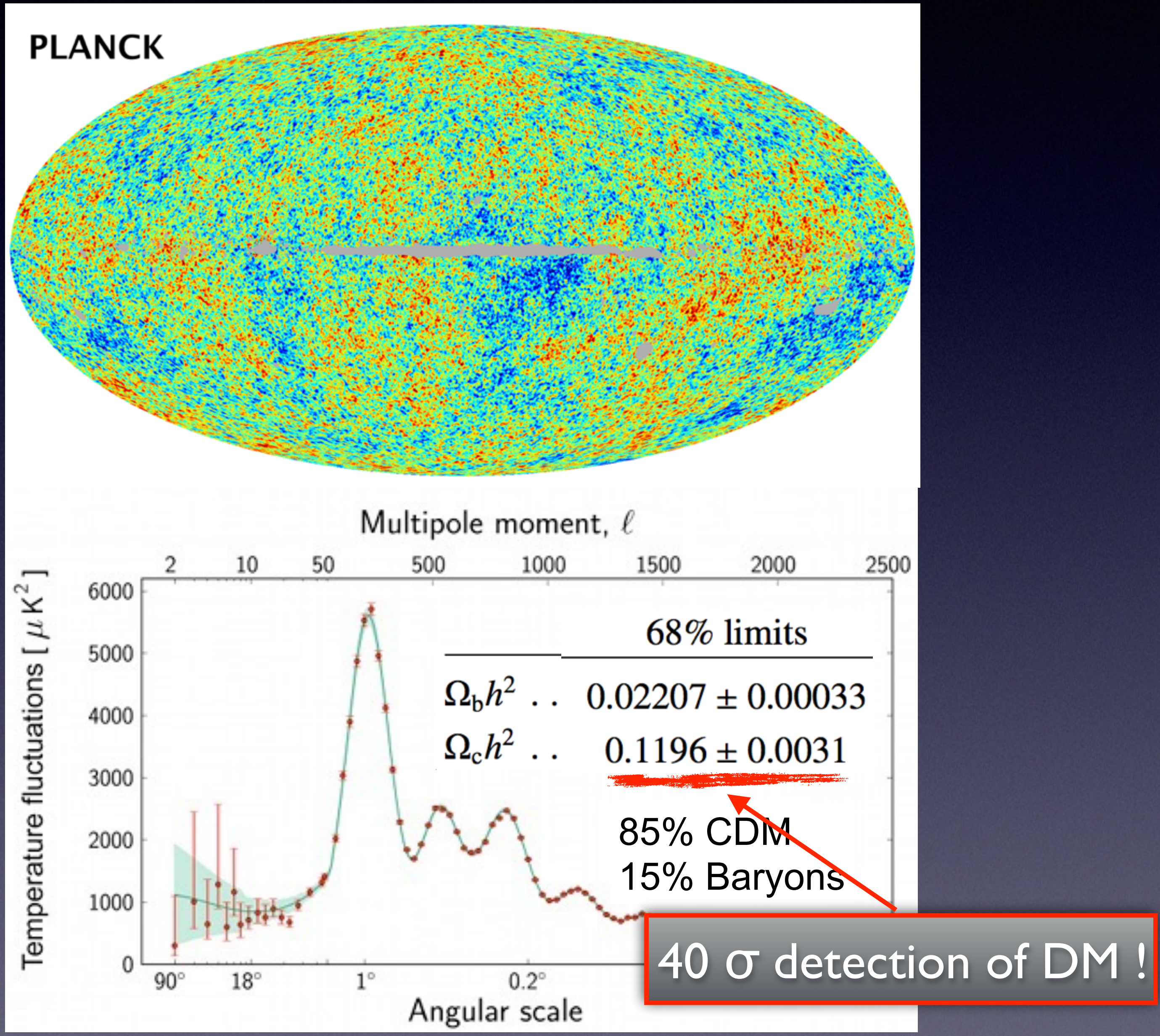
➡ **Probe universal geometry**
- standard candles (Type-Ia-SNe, GRBs?): Flux=Luminosity / ($4 \pi d_L^2$)
- standard rulers (CMB, BAO): Angular size = Physical size / $d_A^2$

➡ **Map in space and time the growth of density fluctuations**:
(evolution of cluster abundance, redshift space distortions, weak lensing tomography)

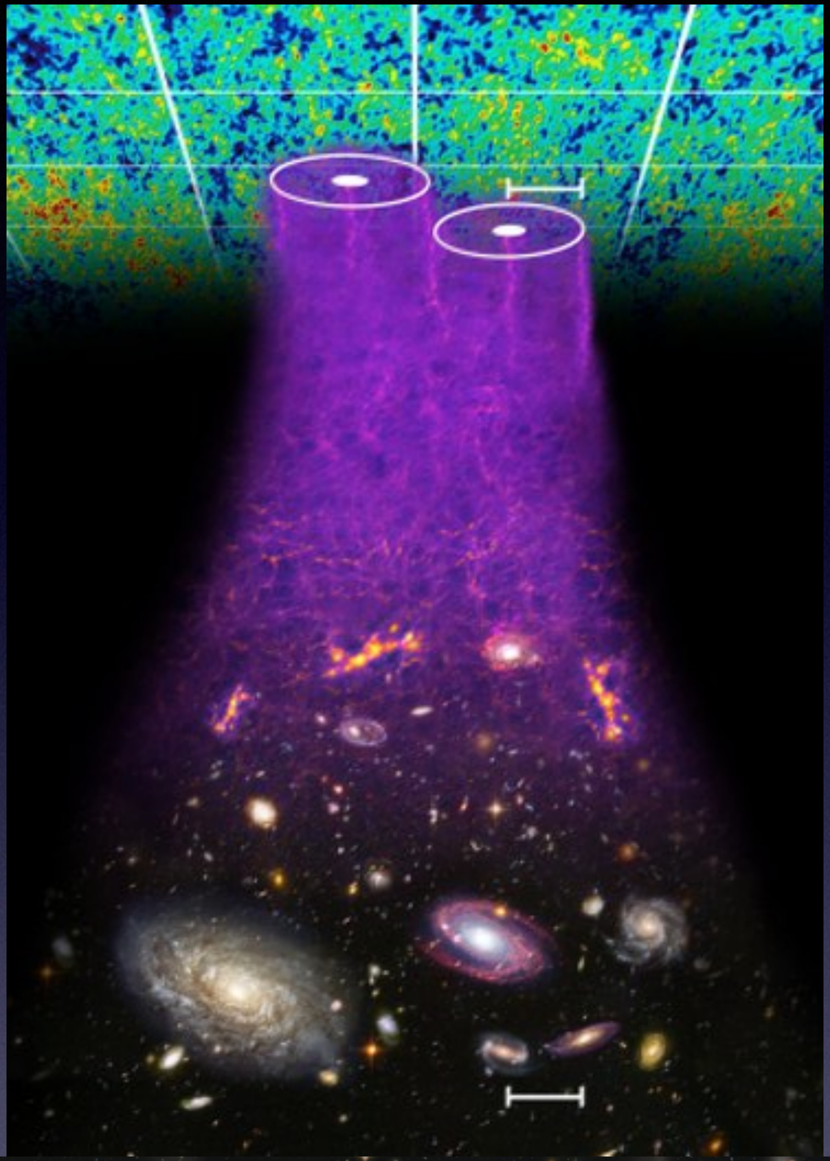
> ➜ **Crucial to disentangle extra $\rho$ components from non-standard gravity**
>
> ➜ **Signature of new physics if they are not consistent**

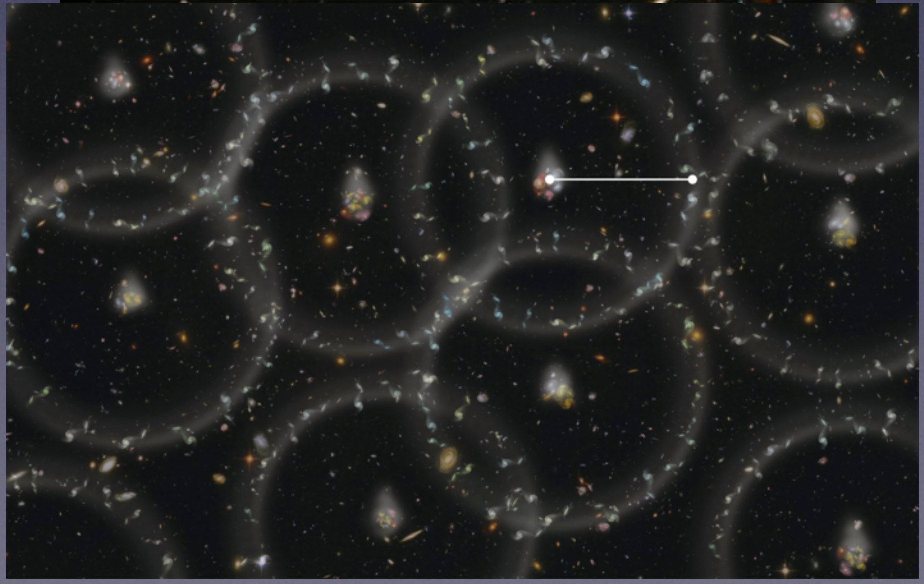
# Measuring the geometry of the Universe
# i.e. its matter content

## Cosmic Microwave Background




Sound horizon (standard rod)

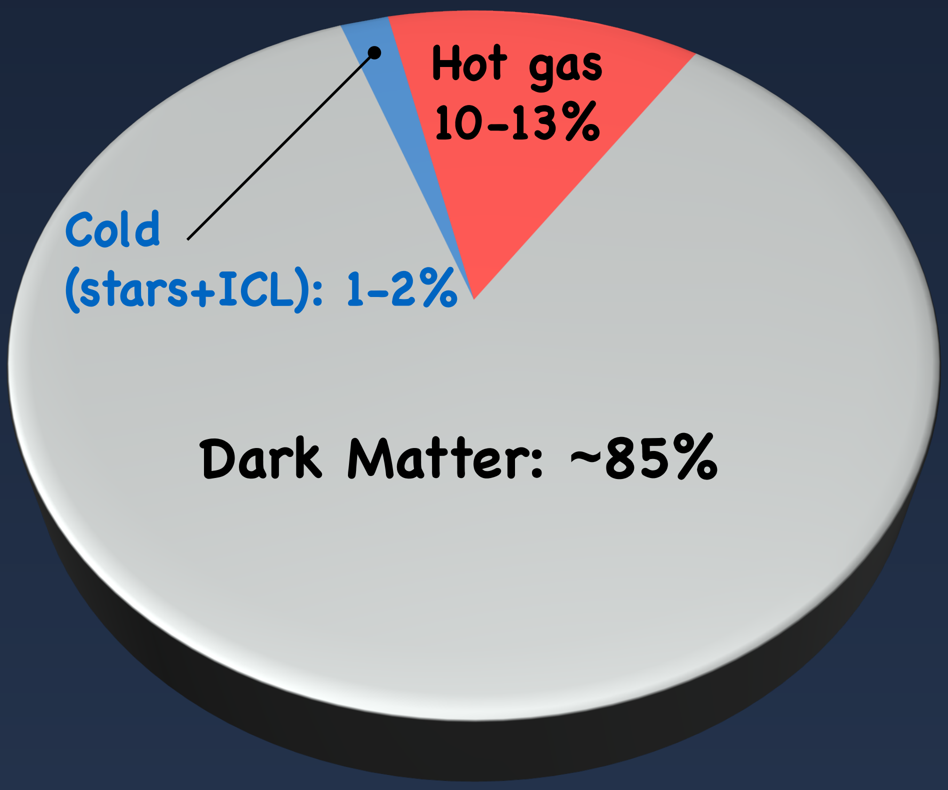




## Baryonic Acoustic Oscillations

# Clusters are powerful probes of structure formation and cosmological models

1) Sensitive probes of the **dark sector** of the Universe (DM+DE)

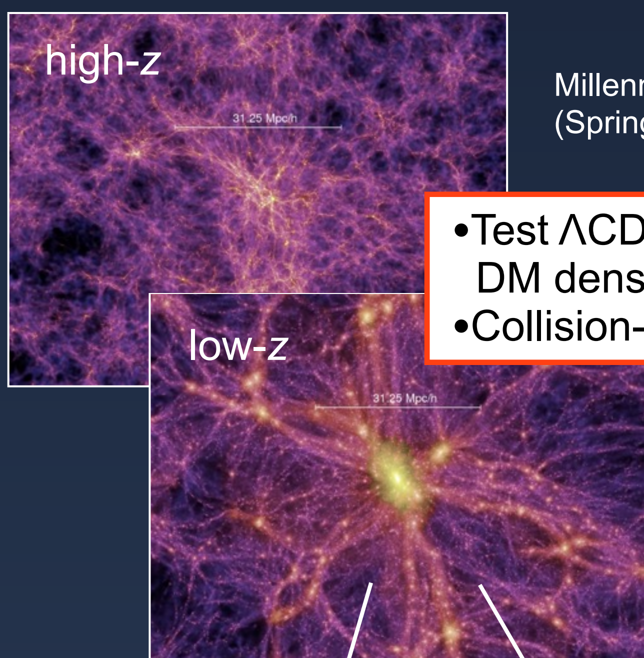



Cluster mass budget

# Clusters are powerful probes of structure formation and cosmological models

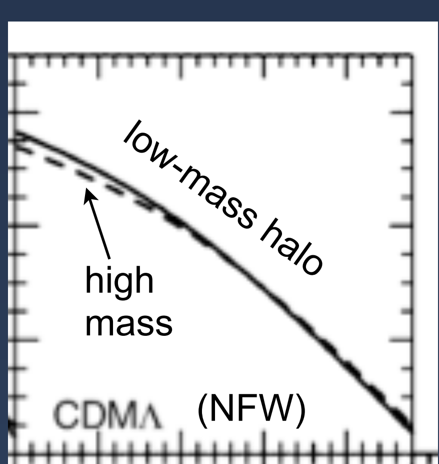
1) Sensitive probes of the **dark sector** of the Universe (DM+DE)

## Structure of DM halos (≲1 Mpc scale)

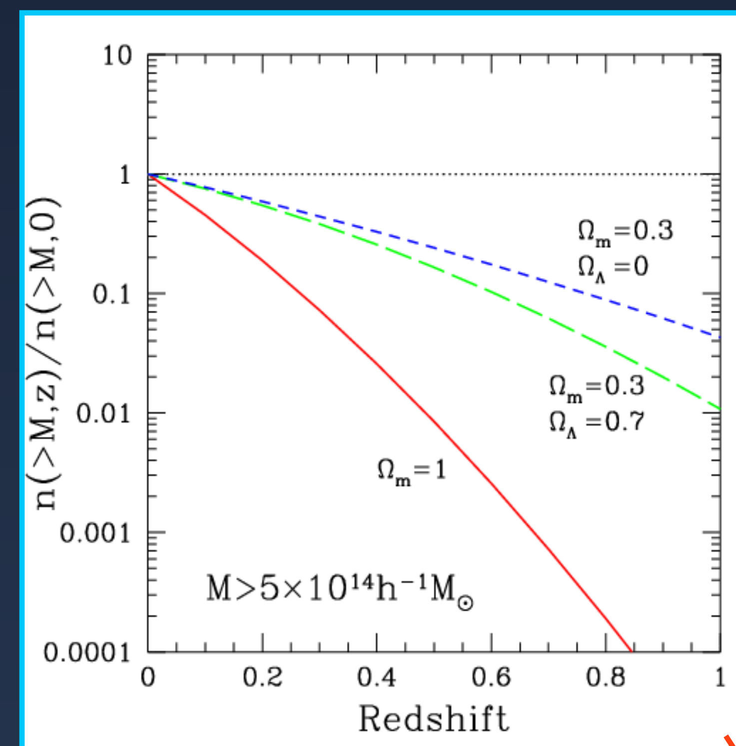


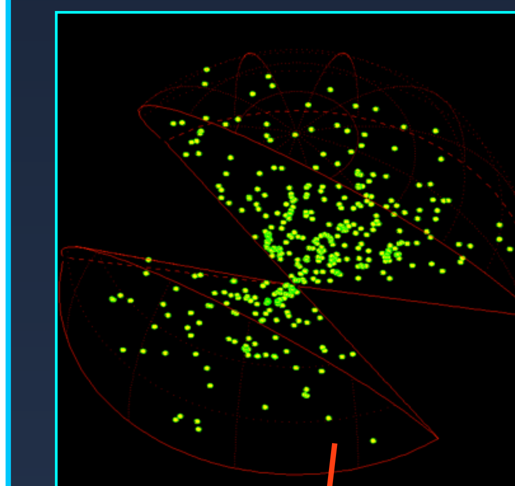

Millennium simulations (Springel et al. 2005)

- Test ΛCDM predictions on DM density profiles
- Collision-less nature of DM?




## Mass function and distribution of DM halos (~ Gpc scale)

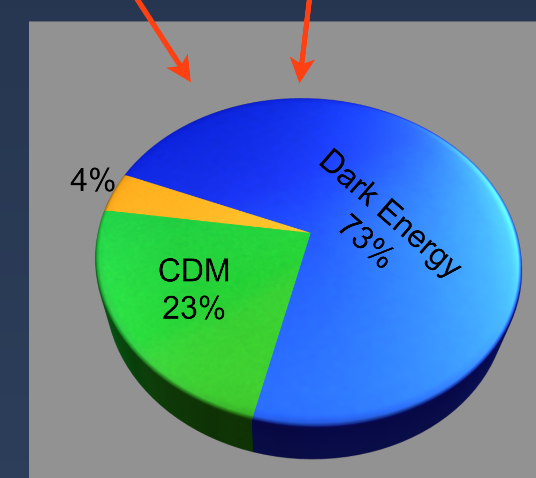


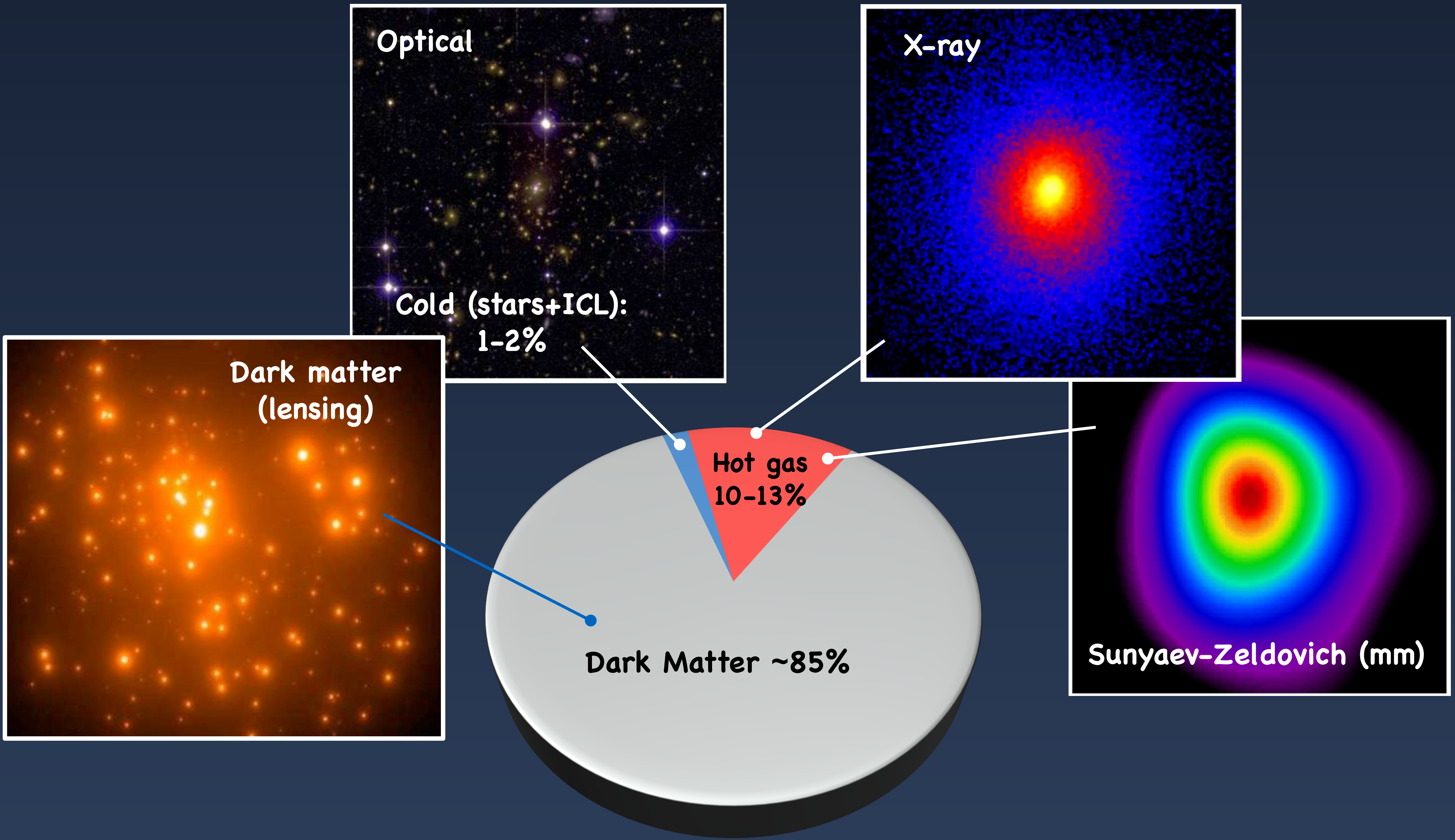
3D distribution

.. depends on
Geometry and Growth:

1. background Cosmology
2. gravity law



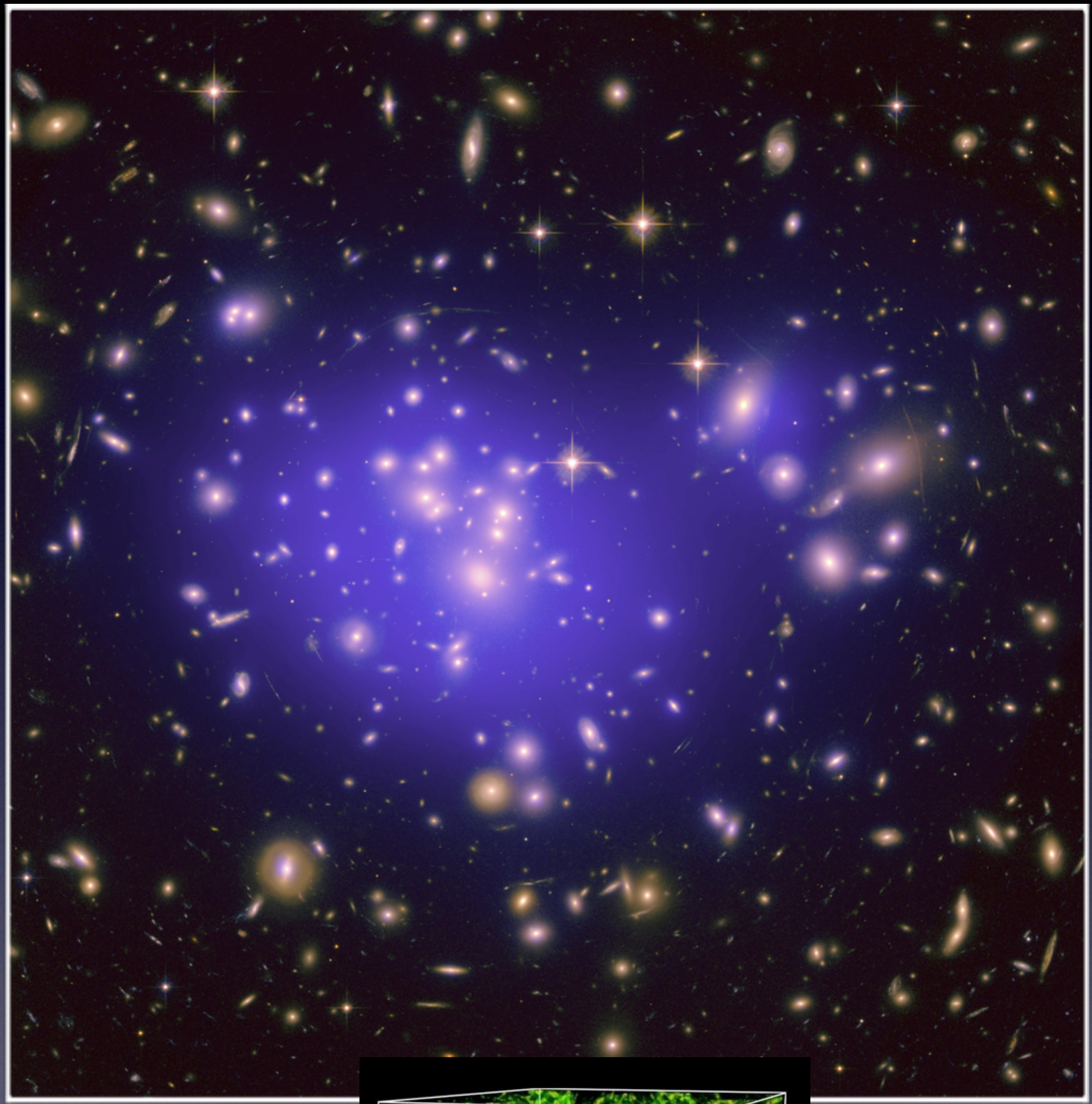
# Clusters are sensitive probes of the dark matter and baryons (cold and hot phase) on large scales




Cluster mass budget

# A simple/robust measurement of $\Omega_M$ (again using galaxy clusters)

- The baryon fraction in clusters can be measured with high accuracy (gas + stars)
- If we have a robust universal measurement of $\Omega_b$ (primordial nucleosynthesis, CMB peaks)
- Then $\Omega_M$ can be readily measured (early 90's !)



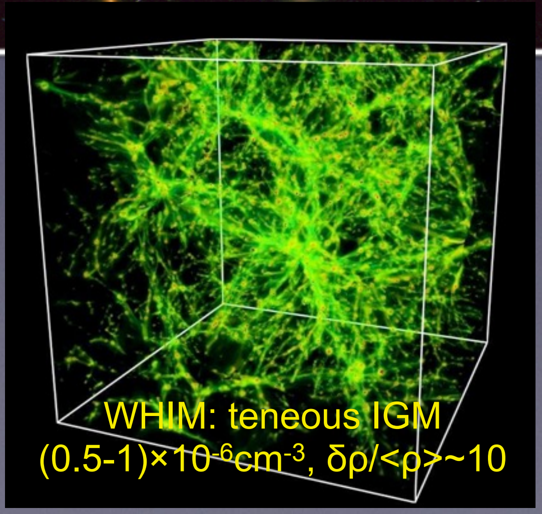
$$f_{bar} = \Omega_b / \Omega_M \quad , \quad f_{bar} = f_{gas} + f_{star} \approx 0.15$$

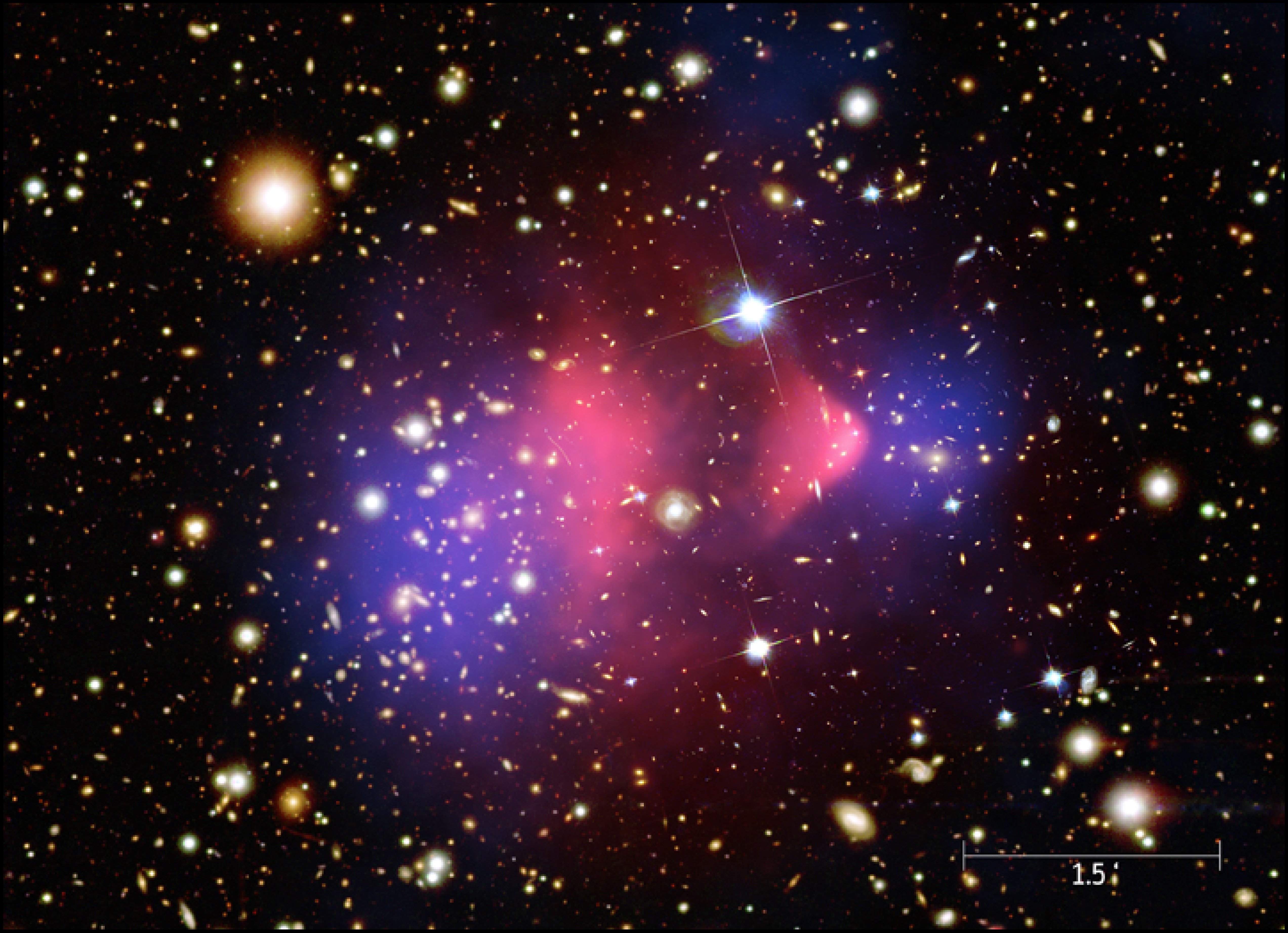
$$\rightarrow \Omega_M = \Omega_b / f_{bar} (= 0.045/0.15) \approx 0.3$$

Note: by summing up all the visible baryons there is a significant fraction of "missing baryons" at z~0 (likely in filaments "WHIM")

$$\Omega_b = 0.045 \pm 0.002 \text{ (for h=0.7, from BBN \& CMB)}$$

$$\Omega_{b,obs@\ z<1} \approx \Omega_{stars} + \Omega_{HI} + \Omega_{H2} + \Omega_{Xray,cl} + \Omega_{Ly\alpha,f} \approx 0.02$$




1.5'

# Mapping normal (baryonic) and dark matter in clusters

- The DM distribution closely follows the one of the galaxies (which behave as collisionless particles (unlike the gas). The lack of “dragging" for DM sets un upper limit to the self-interaction cross-section of DM particles

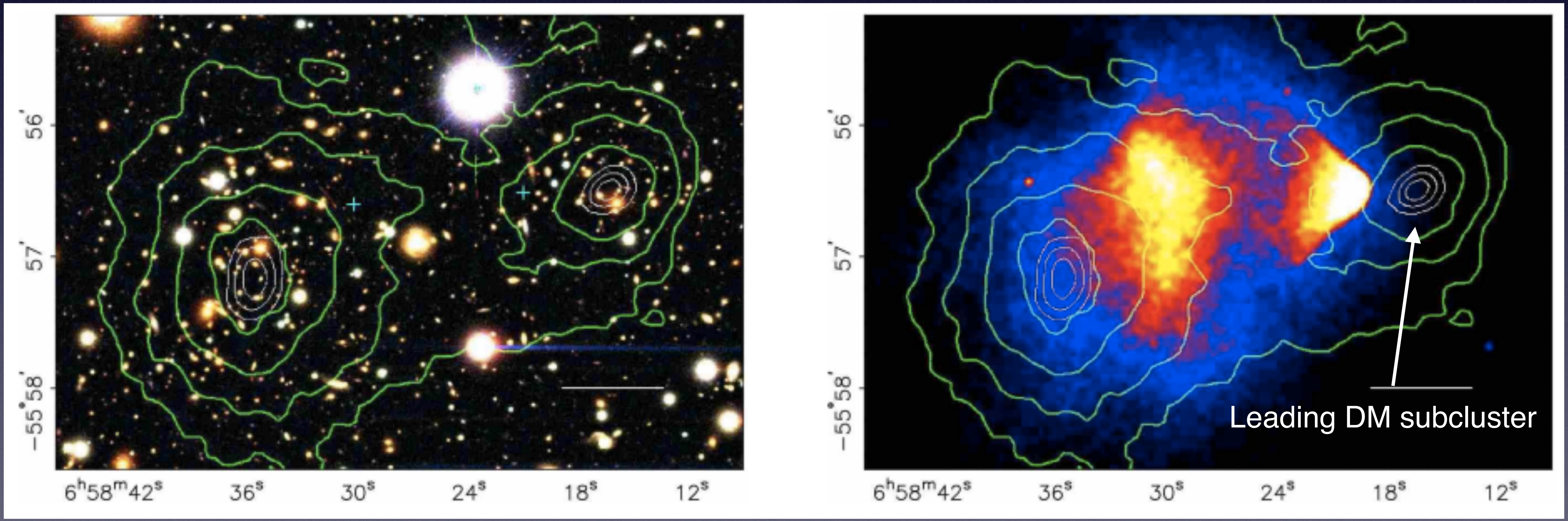
Gas-DM offset implies that subcluster’s scattering depth must be < 1

$$\tau_s = \frac{\sigma}{m}\Sigma_s < 1$$

$$\Sigma_s \simeq 0.2 \text{ g cm}^{-2}$$

DM mass surface density of subcluster from lensing

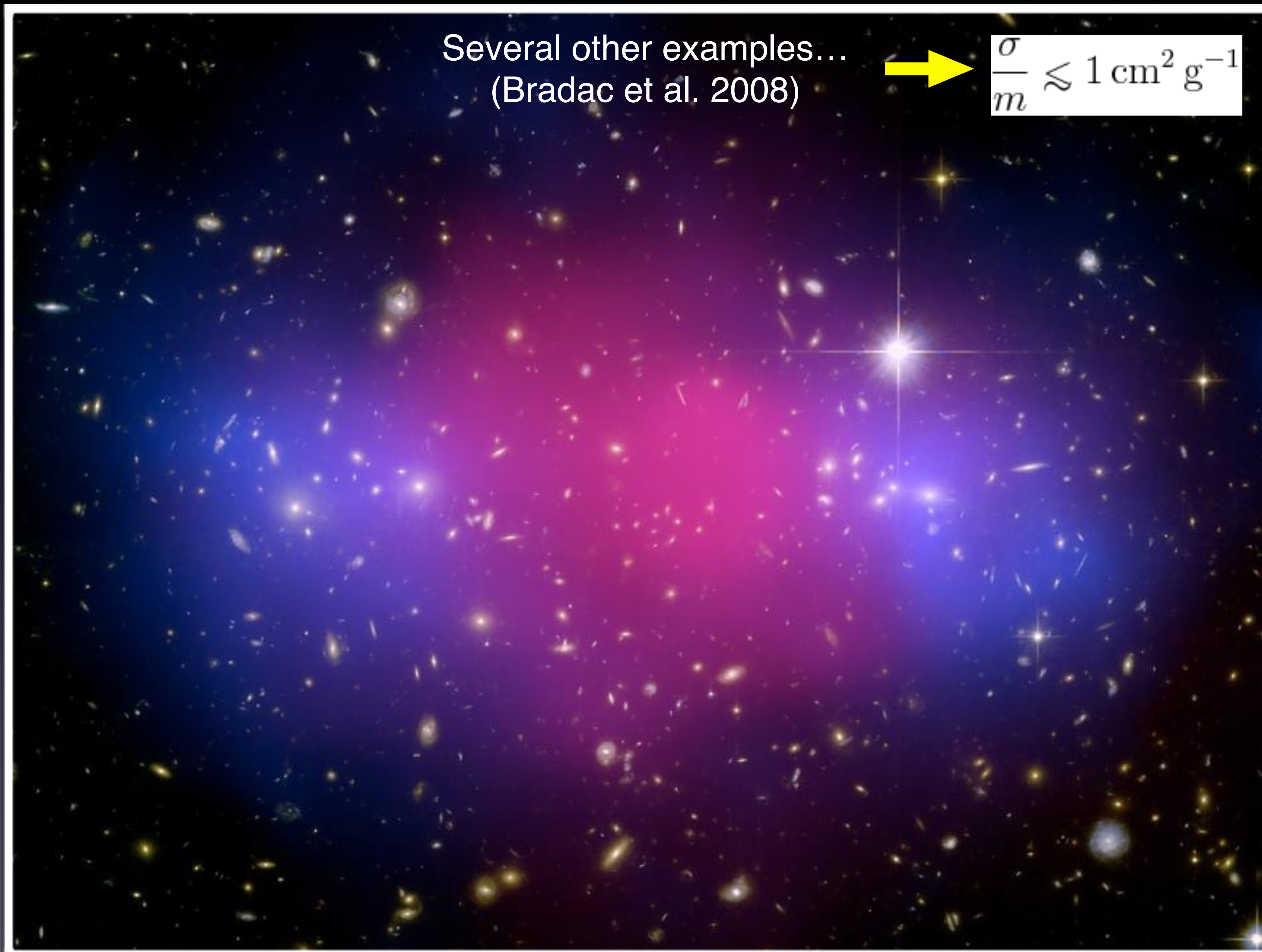
$$\frac{\sigma}{m} < 5 \text{ cm}^2 \text{ g}^{-1}$$




- Other independent methods used to constrain σ/m:
  - High velocity of the leading DM subcluster (4500 km/s)~free fall velocity
  - Survival of the DM subcluster

$$\frac{\sigma}{m} < (1-5)\,\mathrm{cm}^2\,\mathrm{g}^{-1}$$

# Mapping normal (baryonic) and dark matter in clusters



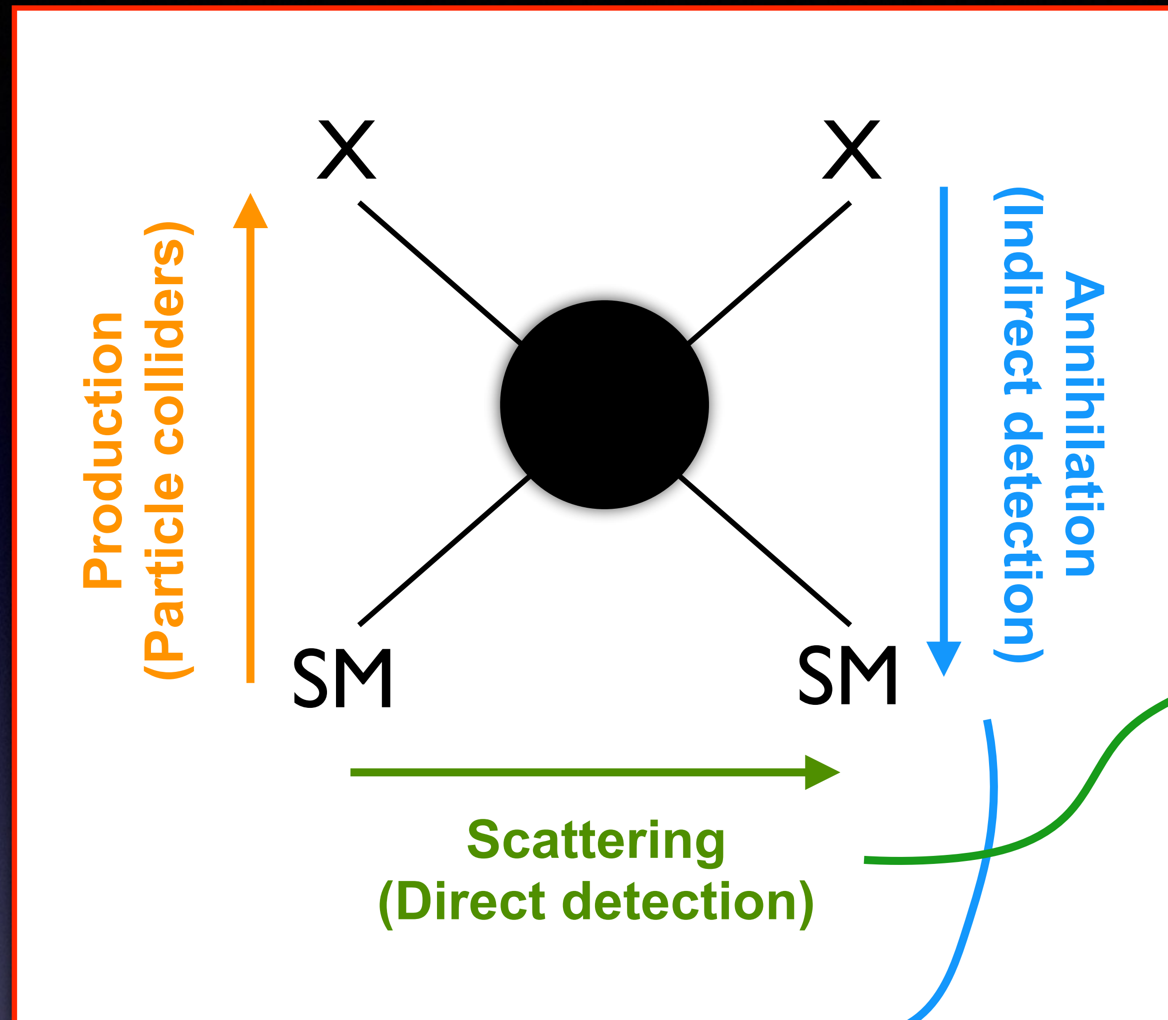

See also recent Harvey et al. Science (sample of "bullet" with HST and Chandra) $\frac{\sigma}{m} < 0.5\ \mathrm{cm}^2\ \mathrm{g}^{-1}$

# Nature of DM particles ???

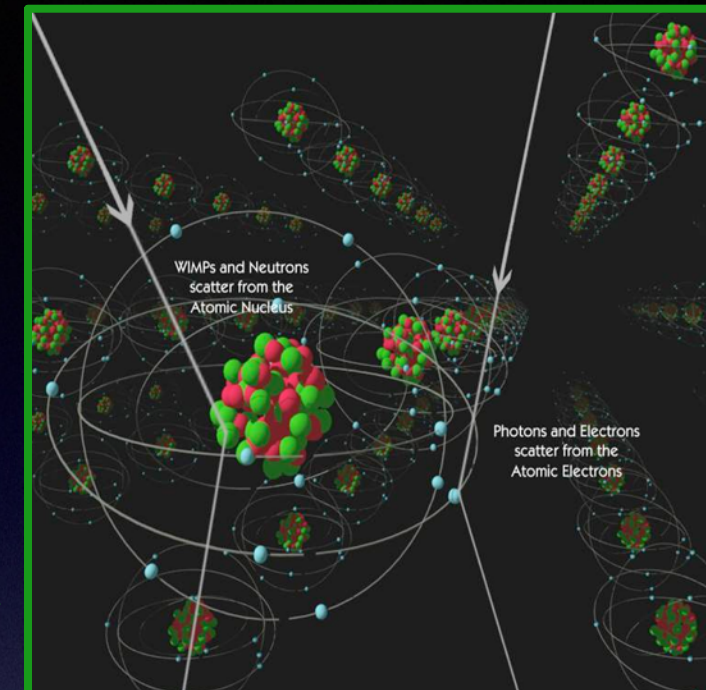
- Cannot be any of the particles we know in the SM
- Has to to be neutral
- Has to be stable over Hubble time
- Has to be non relativistic at decoupling
- If thermally produced thermally in the early Universe their abundance must match the relic abundance ("WIMPS miracle") $\Omega_{\rm DM} h^2 \sim \frac{3 \times 10^{-27} {\rm cm}^3 {\rm s}^{-1}}{\langle \sigma v \rangle}$
- The self-interaction cross section has to be small
- Cross-section and mass have to be within all the existent bounds..
- In principle, it doesn't have to be of just one type..

# Understanding the nature of Dark Matter


$m_X$<~100 GeV



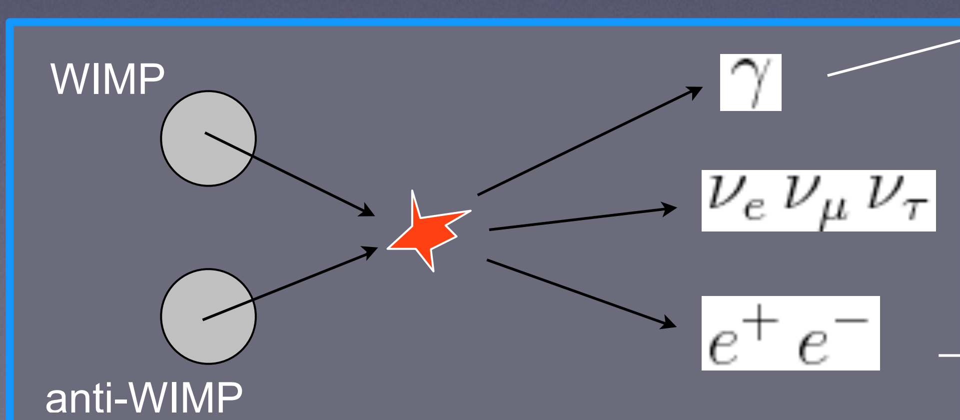

## Direct detection of DM

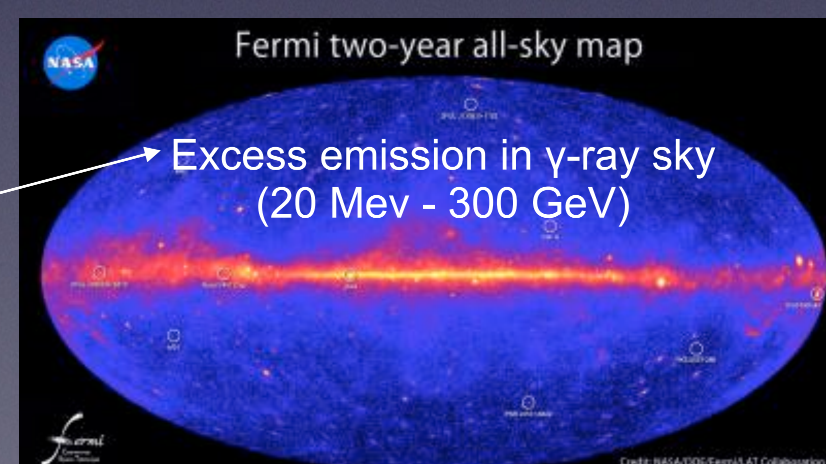



(e.g. CDMS, DAMA, XENON, underground experiments nucleus recoil from WIMP collision)

Detection of recoil energy via ionization (charges), scintillation (light) and heat (phonons)

## Indirect detection of DM (if WIMPs..)



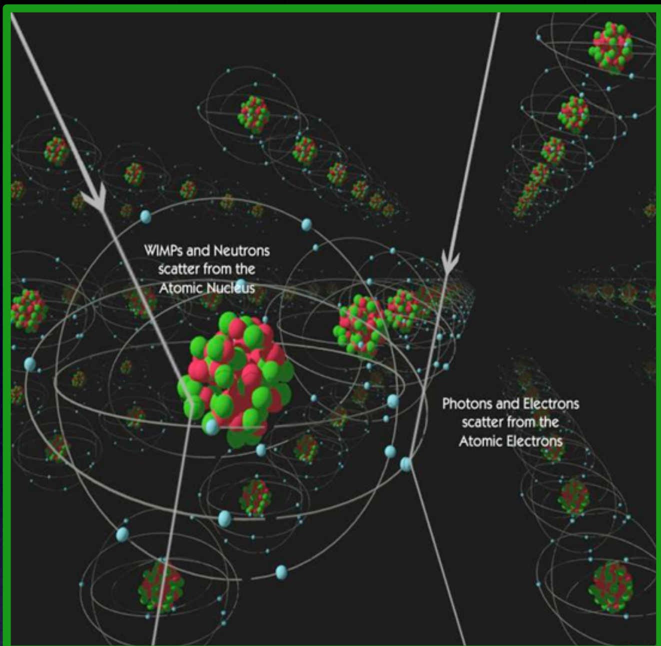

VHE CRs (CTA)

Spectrum of charged positrons and anti-protons (excess of CRs), e.g. Pamela, AMS (no info on source location)

# Understanding the nature of Dark Matter

**Particle physics**

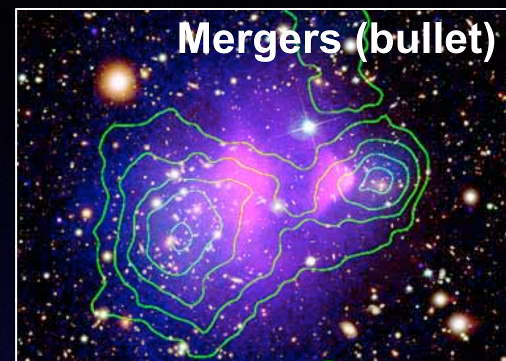
**Direct detection of DM**
(underground experiments)

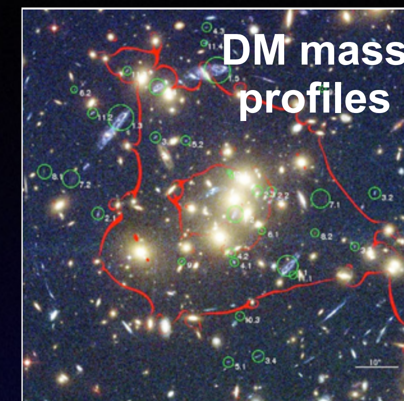




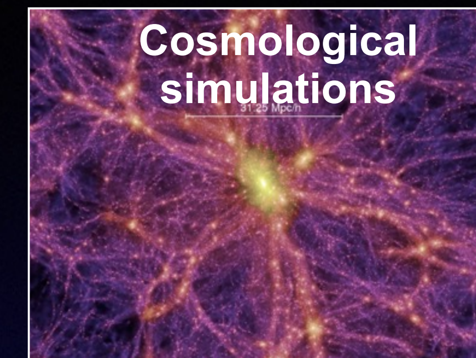
**Astronomy**

**Indirect clues on DM properties from Clusters**




+

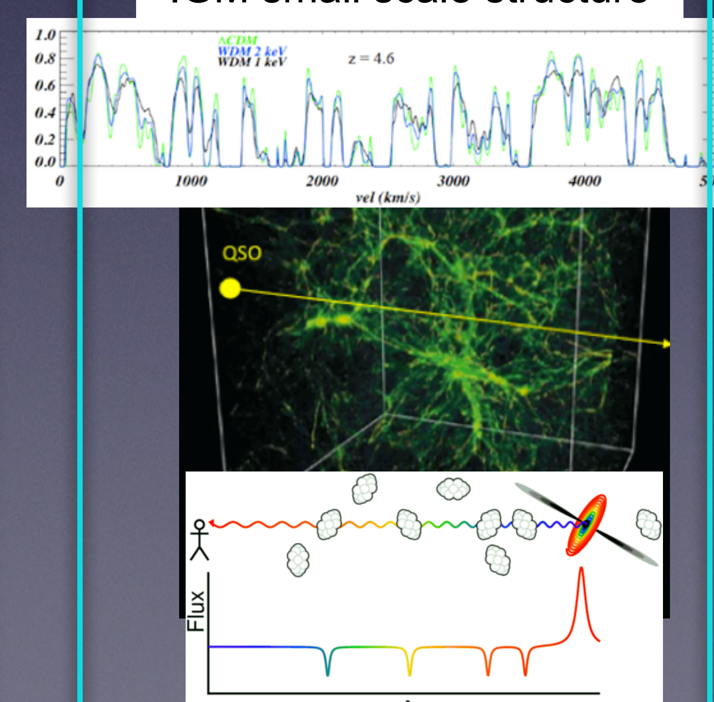



Constraints on self-interacting DM

**Dark Matter particle properties:** $M_X, \sigma_X, \sigma_{X\bar{X}}$
**+**
**Beyond Standard Model**

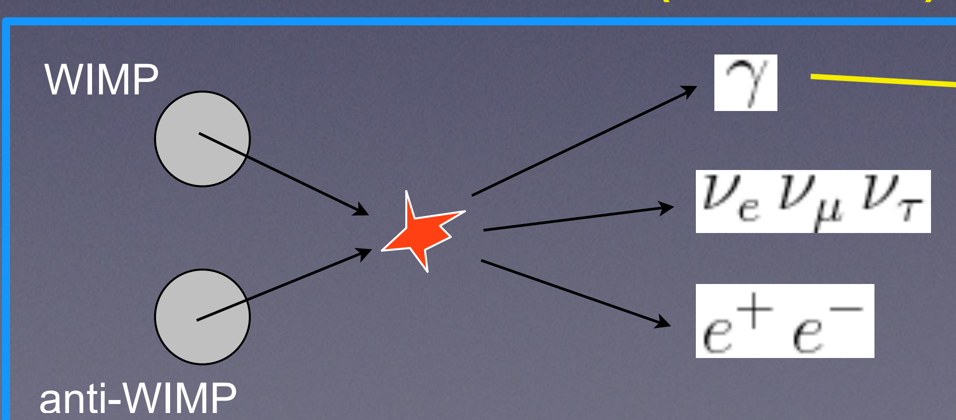
Cold vs Warm DM

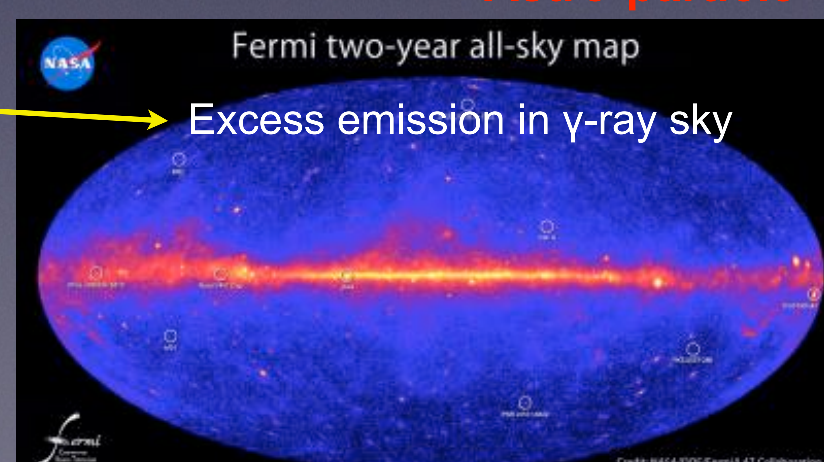

IGM small scale structure




**Indirect detection of DM (if WIMPs..)**

**Astro-particle**

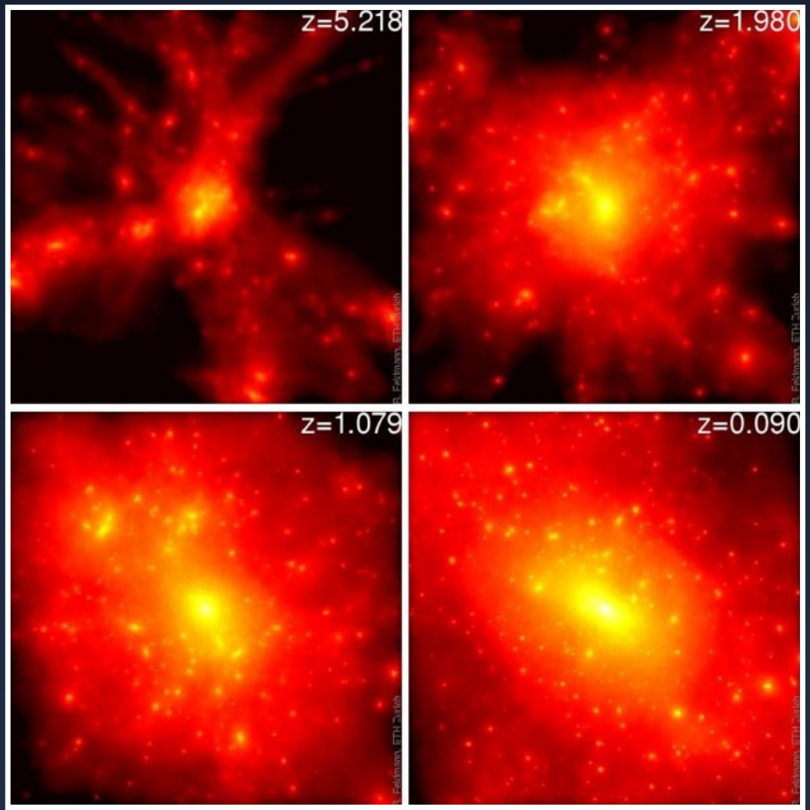


# ΛCDM Predictions for DM Halos

**Hierarchical assembly of CDM halos predicts**:

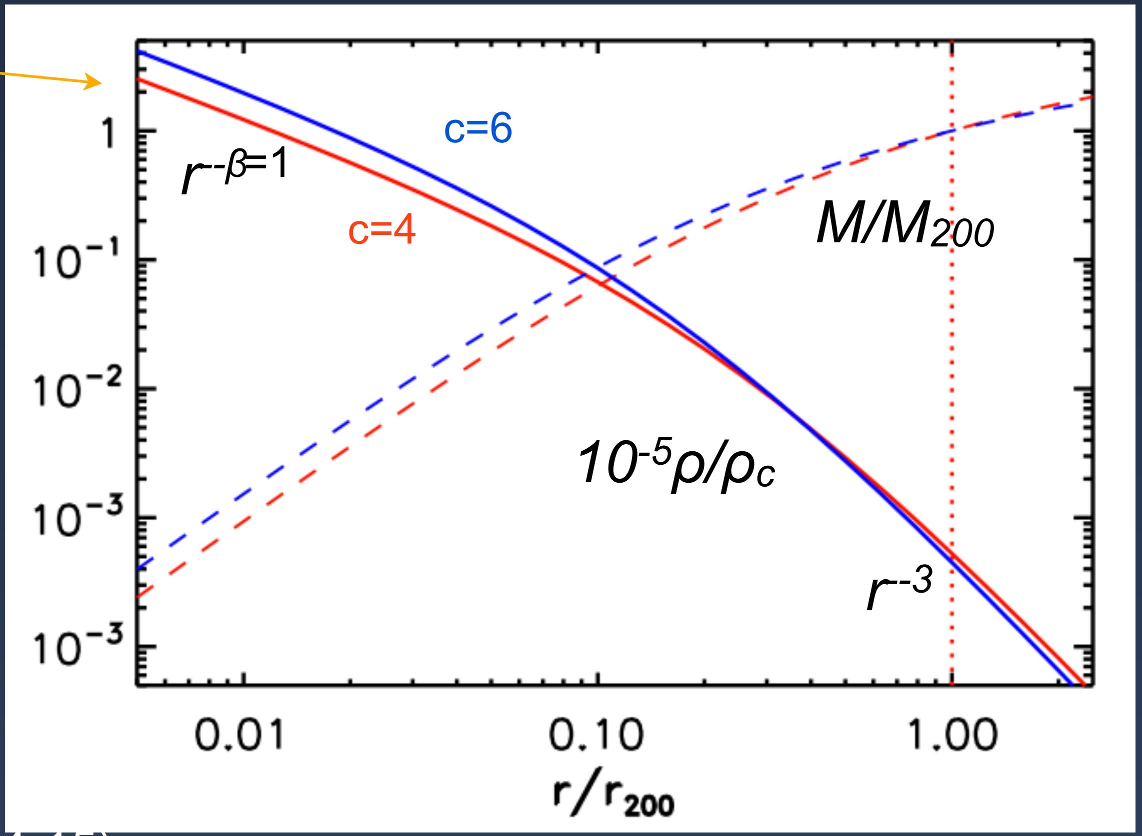
1. mass profiles with a (quasi) universal shape (gals→CL)
2. prominent triaxial shapes
3. "cuspy" inner mass slopes ($\beta \approx 1$)
4. a large degree of substructure
5. halo radial structure result of mass assembly history




(e.g. Navarro+ 97, Duffy+ 08, Gao+ 2008, Bullock+ 11, Klypin+ 2011, Giocoli+ 2012, Bhattacharya+ 2011)

$$\rho(r) = \frac{\rho_S}{(r/r_S)^{\beta}(1+r/r_S)^{(3-\beta)}}$$

**gNFW**




$$\rho_S = \delta_c \rho_{crit}(z) \qquad c \equiv \frac{r_{200}}{r_S}$$

concentration parameter

$c$ depends (mildly) on mass&redshift via the **formation epoch of DM halos**, which depends on the structure formation scenario ➔ **testable prediction of ΛCDM**

$c_{vir} \equiv r_{vir}(M_{vir},z)/r_s(z_{vir})$

(Duffy et al. 08)

$$\bar{c}_{vir} \approx c_0(1+z)^{-A}\left(\frac{M_{vir}}{10^{15}M_{sun}/h}\right)^{-B}$$

Simulations suggest shallow dependence (A,B~0.1-0.3) (Log M=14-15)

# ΛCDM Predictions for DM Halos

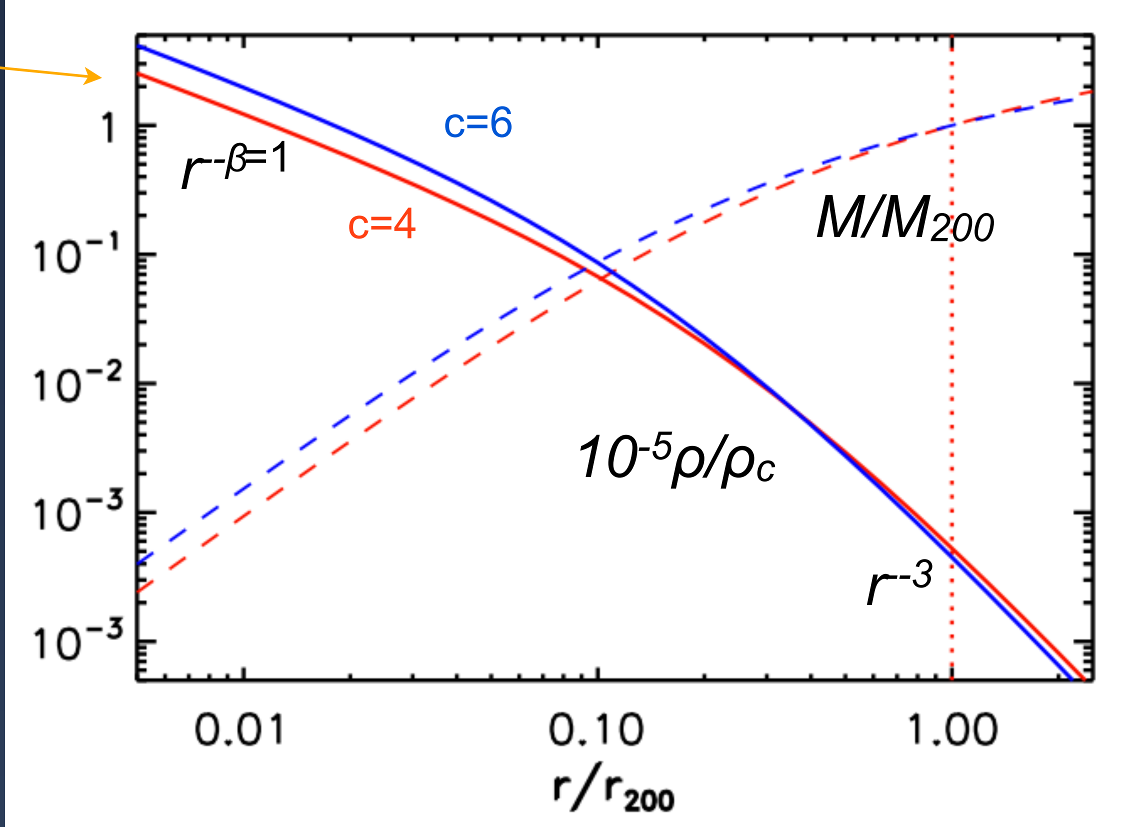
## Highly debated issues

- Concentration-Mass relation: *c(M,z)*
- DM & baryons distribution in the inner core: inner slope of ϱ(r)
  - DM particle physics or dynamical effects of baryons ?
- Degree of substructure of DM halos

(e.g. Navarro+ 97, Duffy+ 08, Gao+ 2008, Bullock+ 11, Klypin+ 2011, Giocoli+ 2012, Bhattacharya+ 2011)

$$\rho(r) = \frac{\rho_S}{(r/r_S)^{\beta}(1+r/r_S)^{(3-\beta)}}$$

**gNFW**




$$\rho_S = \delta_c \rho_{crit}(z)$$

$$c \equiv \frac{r_{200}}{r_S}$$

concentration parameter

*c* depends (mildly) on mass&redshift via the **formation epoch of DM halos**, which depends on the structure formation scenario → **testable prediction of ΛCDM**

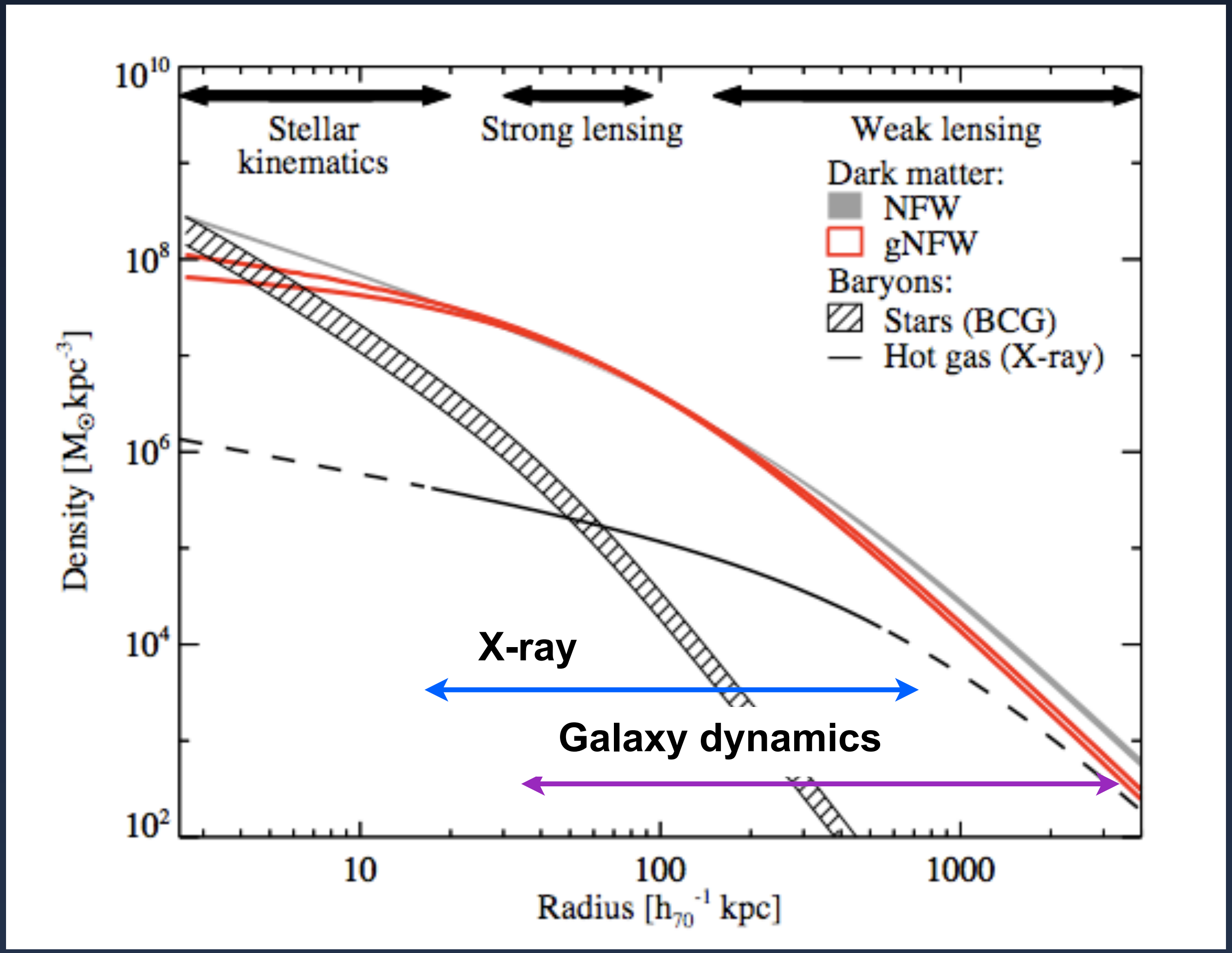
$c_{vir} \equiv r_{vir}(M_{vir},z)/r_s(z_{vir})$

(Duffy et al. 08)

$$\bar{c}_{vir} \approx c_0(1+z)^{-A}\left(\frac{M_{vir}}{10^{15} M_{sun}/h}\right)^{-B}$$

Simulations suggest shallow dependence (A,B~0.1-0.3) (Log M=14-15)

# Measuring DM and Baryon mass density profiles in clusters




Newman et al. 09

- **Key**: use a variety of complementary probes
  - to cover 2-3 decades in scale in a complementary fashion
  - to mitigate systematics (different for each method)
    - Lensing: LSS projections, triaxiality
    - X-ray: deviation from hydrostatic equilibrium, non thermal support
    - Dynamics: deviation from equilibrium, substructures, projections

# X-ray hydrostatic mass for clusters

Balance of gravitational and pressure forces for the gas:

In spherical symmetry: $$\frac{dP}{dr} = -\rho_{gas}\frac{d\Phi}{dr} = -\rho_{gas}\frac{GM(r)}{r^2}$$

$$\frac{dp}{d\rho} = -\rho\frac{GM}{r^2}$$

GAS $p_{gas} = \frac{\rho}{\mu m_p}kT$

DM $p_{DM} = \rho\sigma^2$

$\rho_{gas} \propto \rho_{DM}^{\beta}$ $\quad \beta = \frac{\mu m_p \sigma^2}{kT}$

Note: sound speed $$c_s \approx \sqrt{\frac{P}{\rho_g}} = \sqrt{\frac{nk_B T}{\rho_g}} = \sqrt{\frac{k_B T}{\mu m_p}} \sim 1000\,\mathrm{km\,s^{-1}}$$

then the crossing time $t_{sc} = \frac{2R_A}{c_s} \sim 7\times 10^8\,\mathrm{yr}$, >>$T_{cluster}$ $\Rightarrow$ system in equilibrium

If $\mu$ is the mean molecular weight (avg mass of gas particle in units of proton mass), then $\rho_g = n\langle m\rangle = n\mu m_p$ ; for fully ionized H $\mu = 1/2$; for solar abundance $\mu = 0.63$

*Then for a gas with a mixture of elements:* $p = n\, k_B\, T = \rho_{gas}\, k_B T/\mu m_p$

The total mass profile of a cluster can be computed directly from X-ray observations (imaging and spectroscopy): *$T$, $dT/dr$ , $d\rho_g/dr$*

$$M(<r) = -\frac{r^2}{G\rho_{gas}}\frac{dP}{dr} = -\frac{kT}{G\mu m_p}r^2\left(\frac{d\ln\rho_{gas}}{dr} + \frac{d\ln T}{dr}\right)$$

# Dynamical mass for clusters

For a collisionless system of particles (CDM, galaxies) the equilibrium condition is given by the Jeans equation, which for a non-rotating spherically symmetric system, is:

density profile ↓ radial vel.dispersion profile ↓ anisotropy profile

$$M_J(<r) = -\frac{r\sigma_r^2}{G}\left[\frac{d\ln\nu(r)}{d\ln r} + \frac{d\ln\sigma_r(r)^2}{d\ln r} + 2\beta(r)\right]$$

$\beta = 1 - \frac{\sigma_t^2}{2\sigma_r^2}$ is the orbit anisotropy parameter (β=0 for isotropic velocity field) in terms of radial and tangential vel. disp. components

The observed quantities: projected density profile N(R) and line of sight vel.dispersion profile, $\sigma_{los}(r)$, need to be deprojected with Abell integrals, e.g.

$$\nu(r) = -\frac{1}{\pi}\int_r^\infty \frac{dN}{dR}\frac{dR}{\sqrt{R^2 - r^2}},$$

One can trade M(r) and β(r) ➔ "mass-anisotropy degeneracy" which can be removed with an independent knowledge of M(r)

E.g. MAPOSSt method (Mammon et al.): fit projected phase-space distribution of galaxies for a parametric description of M(r) obeying Jeans equilibrium (so $r<R_{200}$)
➔ fitting params: $r_{200}$ (or $M_{200}$), scale radius($r_s$, $r_{-2}$) and β(r)

---

Analogy with X-ray hydrostatic mass

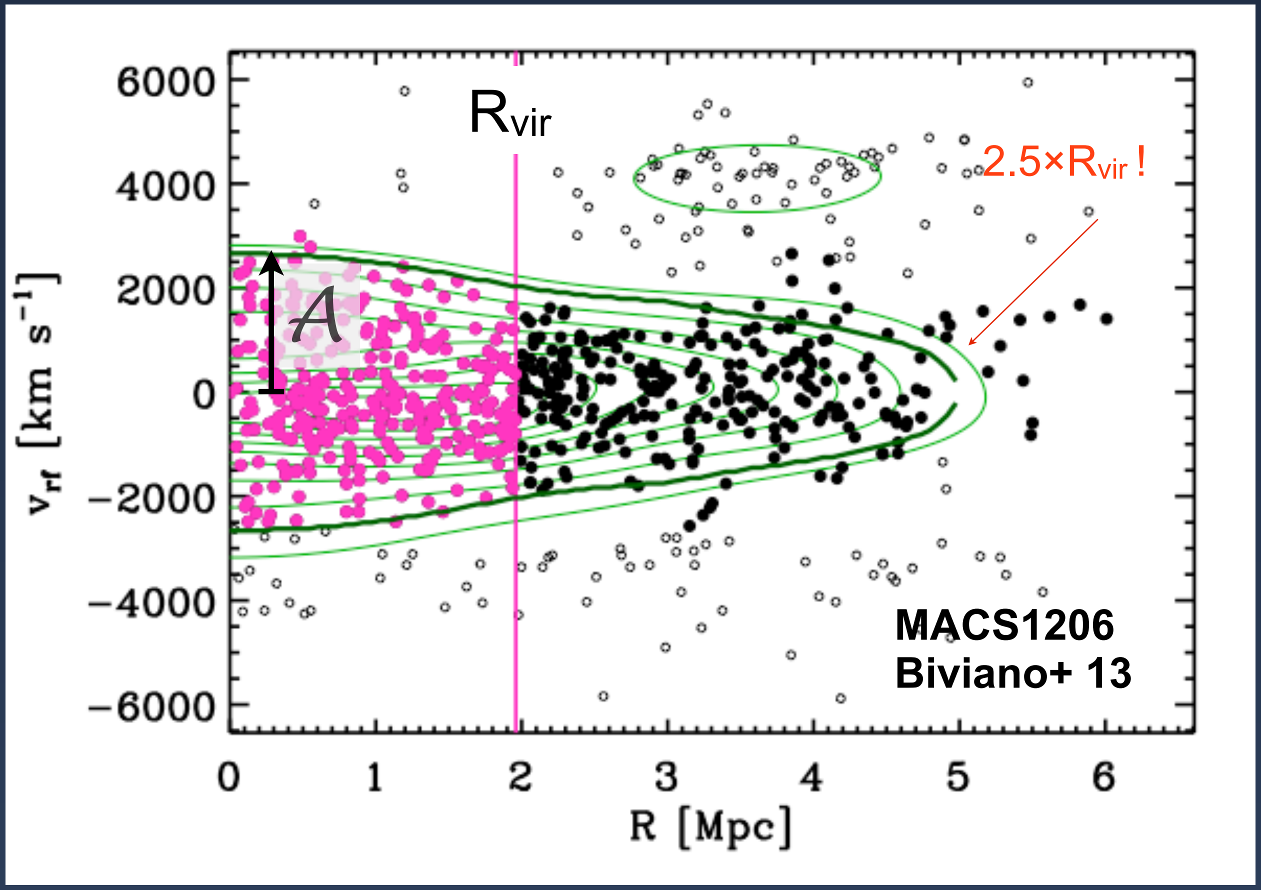
$$M(<r) = -\frac{r^2}{G\rho_{gas}}\frac{dP}{dr} = -\frac{kT}{G\mu m_p}r^2\left(\frac{d\ln\rho_{gas}}{dr} + \frac{d\ln T}{dr}\right)$$

$\rho_{gas} = n\,\mu\,m_p$

$k_B T = \mu\,m_p\,\sigma^2$

# Cluster mass profiles beyond the virial radius ?

- **The Jeans equation can be applied only out to the virial radius ($R_{200}$~1.5-2 Mpc for massive clusters): dynamical equilibrium!**
- **X-ray based masses are often limited to $R_{500}$ (SB limit) and require hydrostatic equilibrium**
- **Weak lensing can in principle be extended beyond $R_{vir}$ but is limited by data depth/quality AND large-scale structure along the line of sight**
- **Kinematics of galaxies beyond $R_{vir}$ can however still probe the cluster potential caustics/phase space method** (Diaferio & Geller 2009)




Amplitude of the caustics $\mathcal{A}(R)$ reflects escape velocity
➔ avg component along the l.o.s. of the $v_{esc}$ at r=R

vel. anisotropy param.

$$v^2_{\rm esc}(r) = -2\phi(r)$$

$$\mathcal{A}^2(r) = \langle v^2_{\rm esc,los} \rangle \Rightarrow \quad -2\phi(r) = \mathcal{A}^2(r) g(\beta)$$

$$GM(<r) = \int_0^r \mathcal{A}^2(r)\mathcal{F}_\beta(r){\rm d}r \simeq \mathcal{F}_\beta \int_0^r \mathcal{A}^2(r){\rm d}r$$

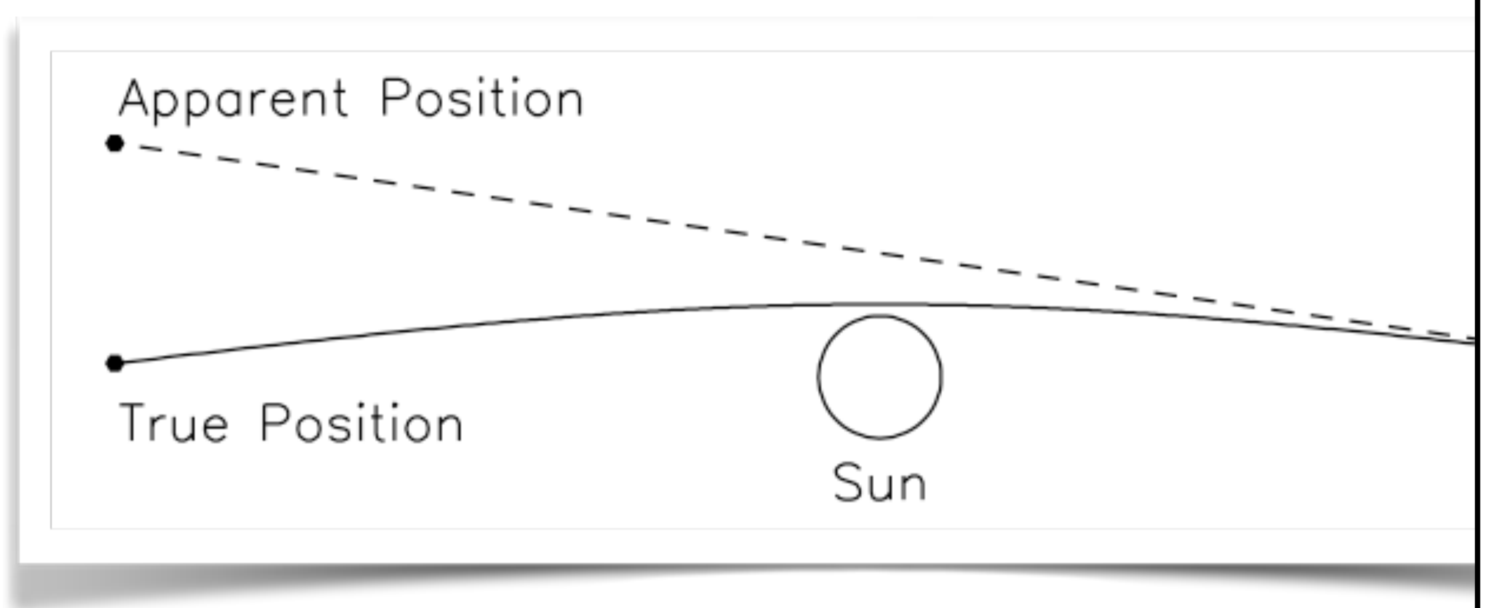
- Does not require assumption of dynamical equilibrium
- All galaxies even beyond $R_{vir}$ can be used
- M(<R) can be determined at R>$R_{vir}$ in a model indep. way, but systematics due to approximation on $\mathcal{F}_\beta(r)$

# Gravitational Lensing: brief historical perspective

- Hypothesis of light deflection by Newtonian gravity goes back to Newton and Laplace, Soldner (1804) derives the classical deflection formula

$$\alpha = \frac{2GM}{c^2}\frac{1}{r}$$

- Einstein (1915) using GR equations finds a deflection angle with a factor of 2 higher than the classical formula (1.74” for the Sun)
- Eddington (1919) confirms the deflection prediction of stars near the solar limb




LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BO NYT 1919 E MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

# Gravitational Lensing: brief historical perspective

- Hypothesis of light deflection by Newtonian gravity goes back to Newton and Laplace, Soldner (1804) derives the classical deflection formula

$$\alpha = \frac{2GM}{c^2}\frac{1}{r}$$

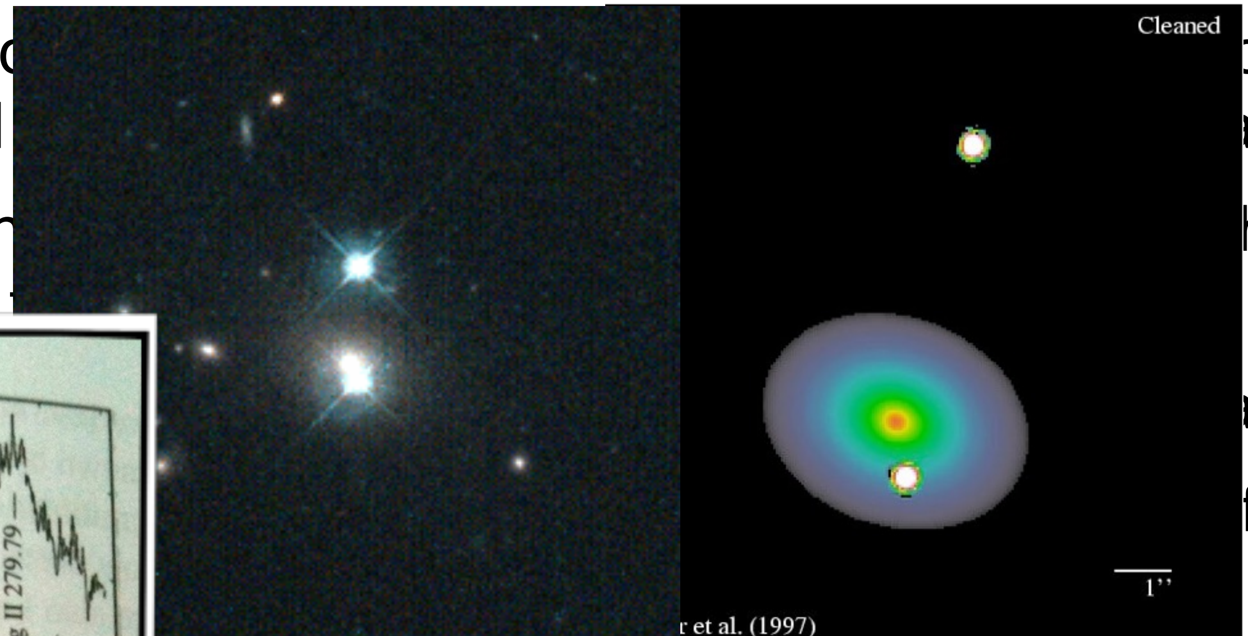
- Einstein (1915) using GR equations finds a deflection angle with a factor of 2 higher than the classical formula (1.74" for the Sun)
- Eddington (1919) confirms the deflection prediction of stars near the solar limb
- Chwolson (1926) conceives the possibility of multiple images ("fictitious stars") of stars by a lensing stars, and even rings in symmetric geometry
- Einstein (1936) considers the same possibility (also rings) and concludes there is no chance to observe the effect for stellar-mass lenses..
- Zwicky (1937) using his new galaxy mass estimates (~4×11 $M_{\odot}$) concluded:
  - –lensing by galaxies can split images to large observable angles
  - –this could be used to estimate galaxy masses
  - –magnification can lead to access distant faint galaxies!
- Refsdal (1964): time delay from variability of multiple sources can be used to measure $H_0$ (if an accurate mass model is available..)

# Gravitational Lensing: brief historical perspective

- Hypothesis of light c … o Newton and Laplace, Soldner (1 … a

$$\alpha = \frac{2GM}{c^2}\frac{1}{r}$$

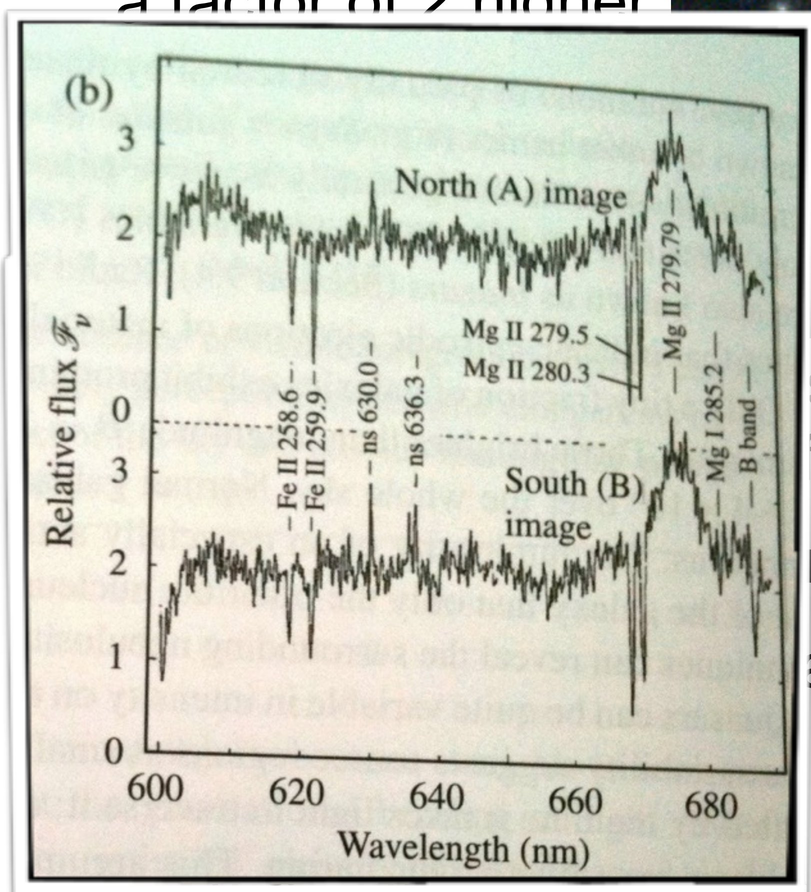
- Einstein (1915) usin … h a factor of 2 higher … )




… r the solar limb

… fictitious stars") of

… s the same possibility (also rings) and concludes there is no … ffect for stellar-mass lenses..

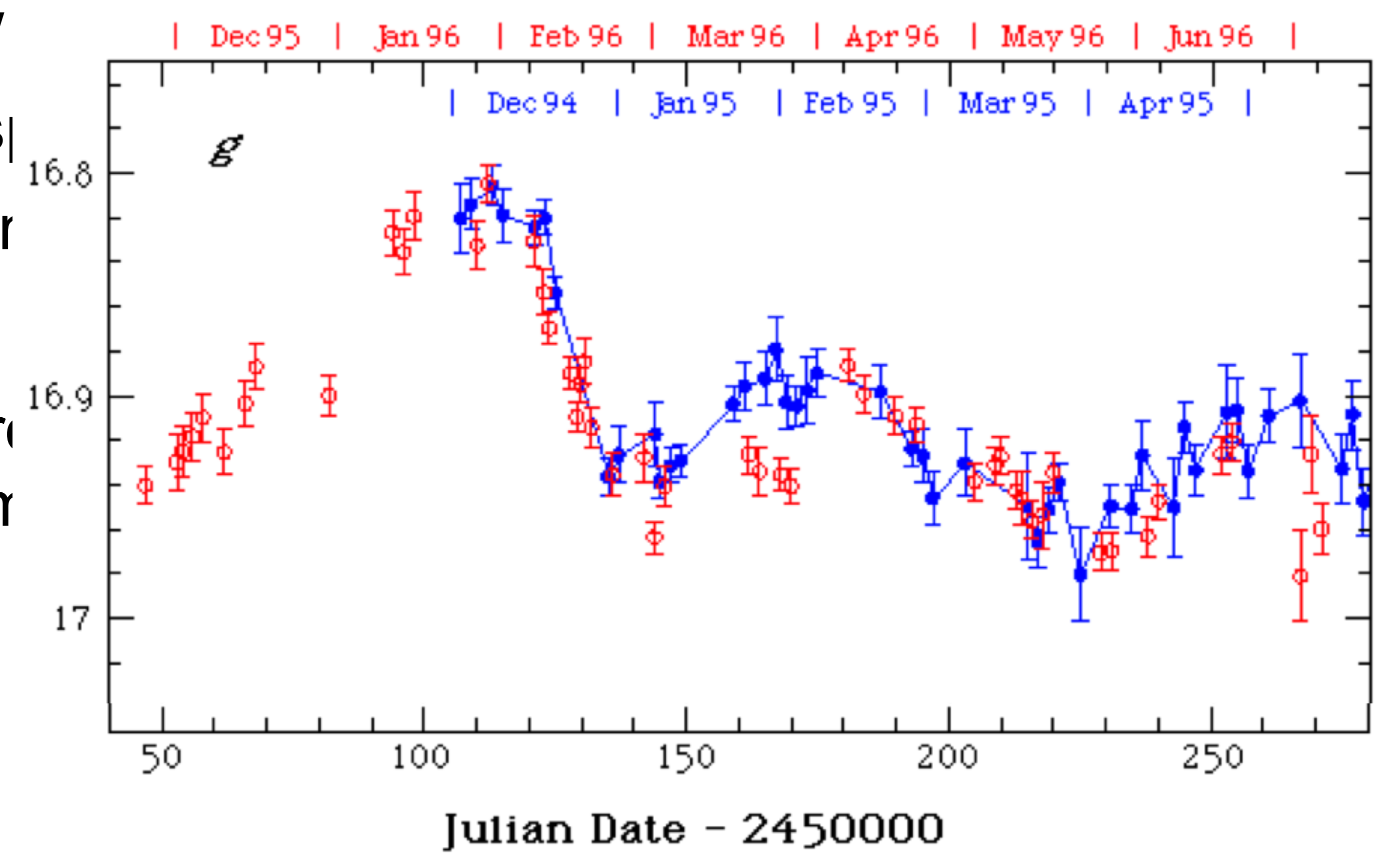



… new

… an s

… esti

… d to




- Refsdal (1964): time delay fr … measure $H_0$ (if an accurate m …
- Walsh et al. (1979) discover …

# Gravitational Lensing: brief historical perspective

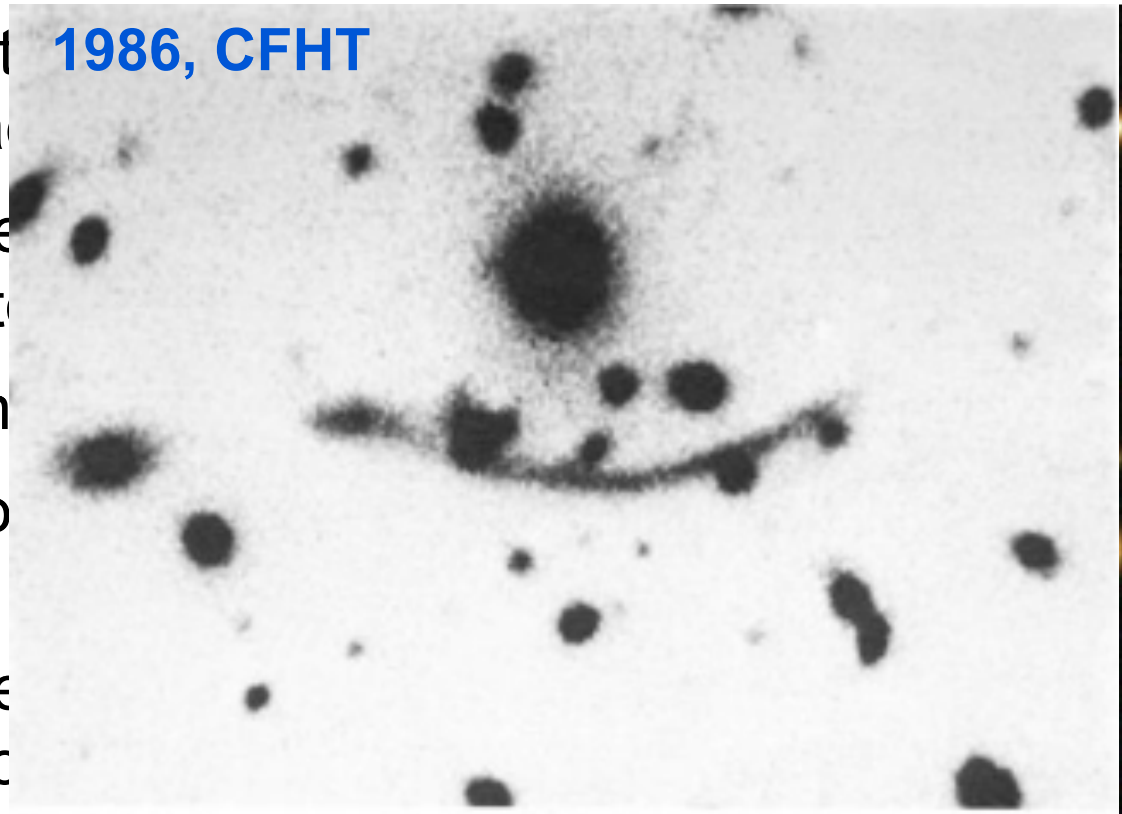
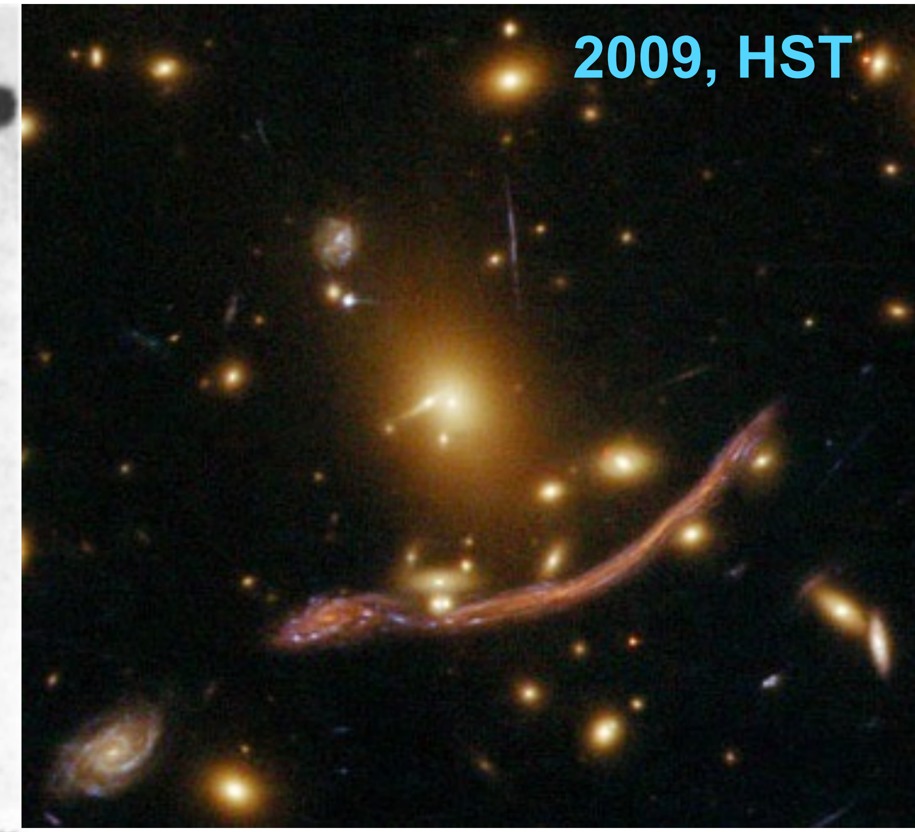
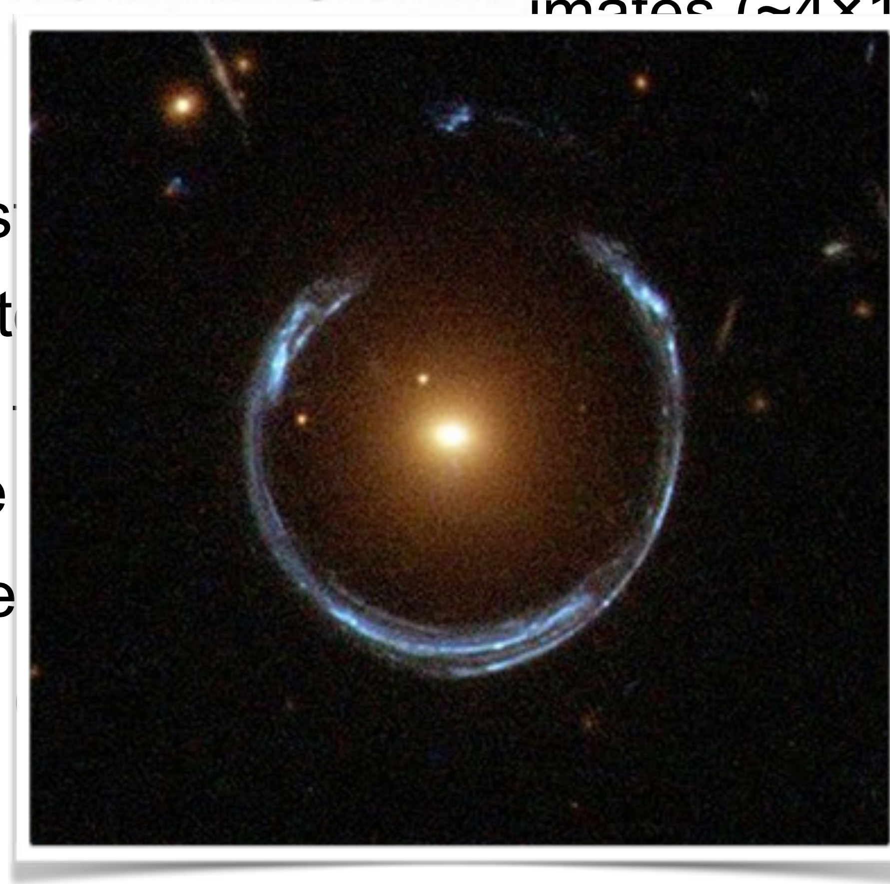



Figure 2: The giant luminous arc in Abell 370. CFHT, 0.″2/pixel, 10 min., seeing 0.″7, November 25, 1986.

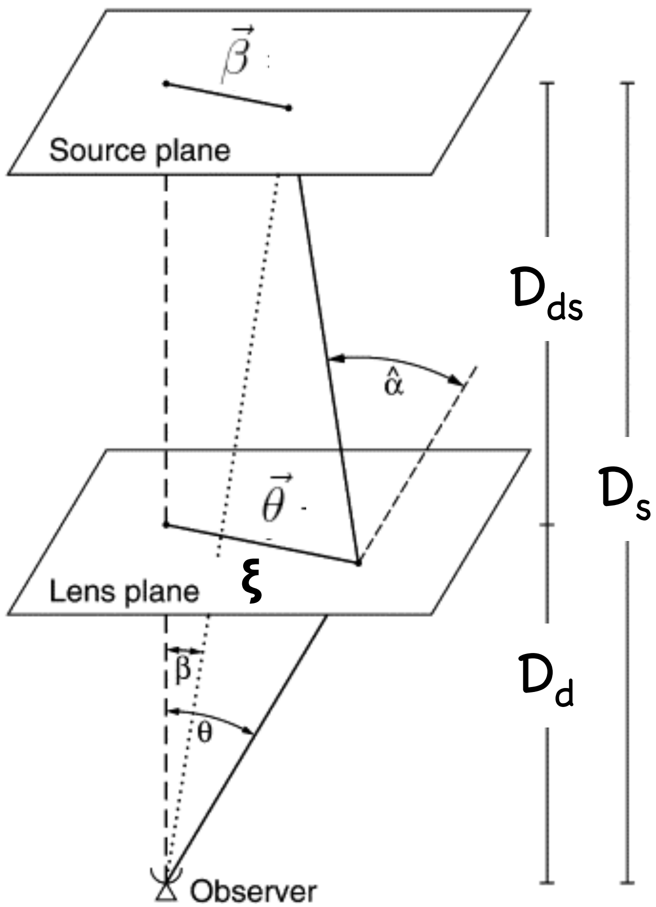



$$\alpha = \frac{2GM}{c^2}\frac{1}{r}$$

- Hypot
  Lapla
- Einste
  a fact
- Eddin … b
- Chwo … ) of
  stars
- Einste … e is no
  chanc
- Zwick … imates (~4x11 $M_\odot$) concluded:
  - –lensing by galaxies can … e angles
  - –this could be used to es
  - –magnification can lead t
- Refsdal (1964): time delay … es can be used to
  measure $H_0$ (if an accurate
- Walsh et al. (1979) discove … rt)
- First giant arcs discovered … 7): right interpretation



# Lensing basics




- A lens is fully characterized by its surface mass density $\Sigma(\theta)$, or $K(\theta)= \Sigma(\theta)/\Sigma_{cr}$ (*convergence*), $\Sigma_{cr} = \frac{c^2 D_s}{4\pi G D_d D_{ds}}$

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$ Lensing equation

$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla}\psi = \frac{1}{\pi}\int \kappa(\vec{\theta}')\frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2} d^2\theta'$$ deflection field $\boldsymbol{\alpha}(\boldsymbol{\theta}) := \frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}}(D_d\boldsymbol{\theta})$

Sum of all deflections due to all mass elements dm=Σ ds=Σ d²ξ

$$\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi}) = \frac{4G}{c^2}\int d^2\xi' \, \Sigma(\boldsymbol{\xi}')\frac{\boldsymbol{\xi}-\boldsymbol{\xi}'}{|\boldsymbol{\xi}-\boldsymbol{\xi}'|^2}$$

$$\hat{\alpha} = \frac{4\,G\,M}{c^2\,\xi}$$ point-like mass

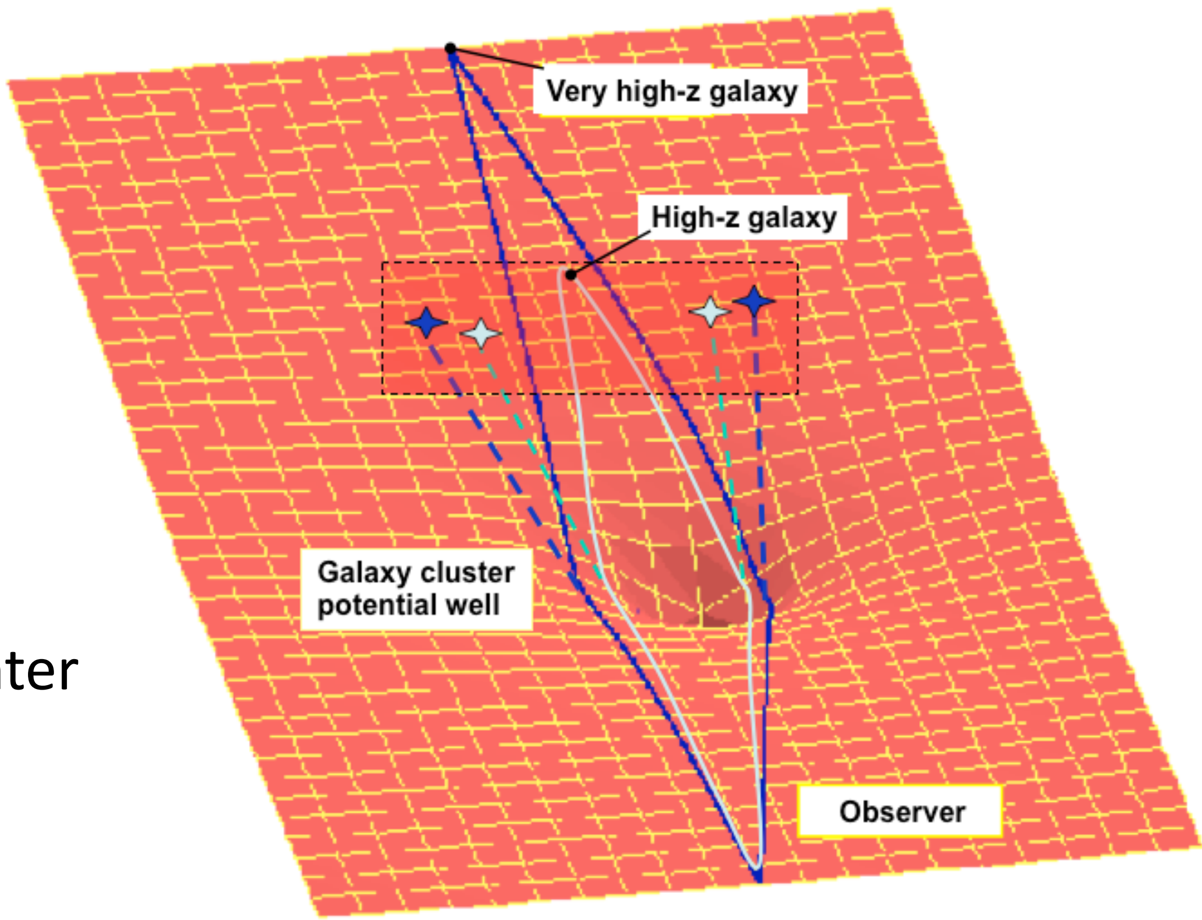
**Einstein radius** ➔ scale of lensing/multiple images

- For circularly symmetric (supercritical) lens with a mass profile $M(\theta)$, an on-axis ($\beta$=0) source is imaged as ring with radius $\theta_E$

$$\theta_E = \left[\frac{4GM(\theta_E)}{c^2}\frac{D_{ds}}{D_d D_s}\right]^{1/2}$$

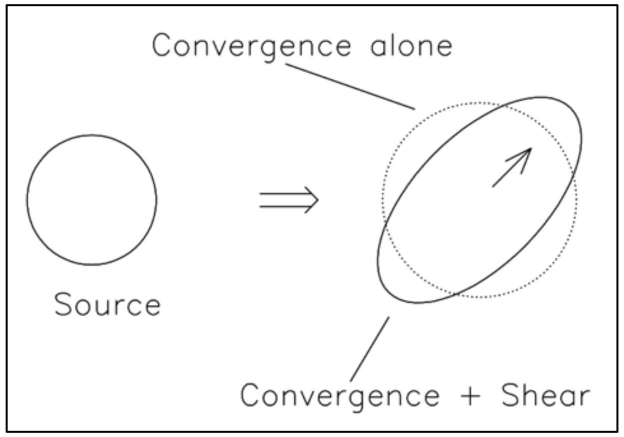
Lensing mapping involves:

- Universal geometry ($\Omega_M$, $\Omega_\Lambda$)
- Lens geometry ($z_L$, $z_S$)
- Cluster mass distribution




- More distant galaxy is imaged further from cluster center
- Geometric lensing deflections can further constraint source redshift

# Convergence and Shear




*convergence* magnifies the image isotropically, the *shear* deforms it to an ellipse (anisotropic part of the lens mapping)

Jacobian matrix $\mathcal{A}$ of the lens mapping $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

$$\mathcal{A} \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left( \delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left( \delta_{ij} - \frac{\partial^2 \psi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right)$$

$$\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$$ magnitude of the shear

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22})$$ *convergence* isotropic term

Under the transformation **β**=$\mathcal{A}$ **ϑ**, a circular object gains an ellipticity *(a-b)/(a+b)* of:

$g = \gamma/(1-\kappa)$ (*reduced shear*), with *magnification*:

$$\mu = \frac{F_I}{F_S} = \frac{\delta\theta^2}{\delta\beta^2} = \frac{1}{\det \mathcal{A}} = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

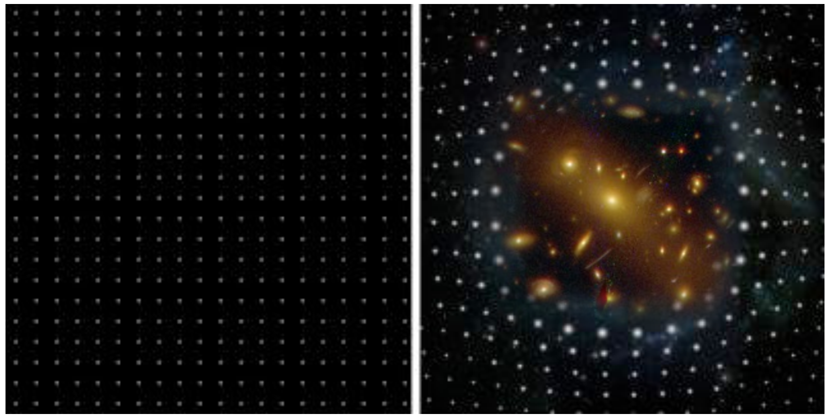
surface brightness is conserved, both galaxy fluxes and sizes are amplified

det $\mathcal{A}$(**ϑ**) = 0 ➔ *critical curves*

**Mass-sheet degeneracy**:

Any reconstruction method is insensitive to isotropic expansions of images
➔ the measured ellipticities are invariant under $\mathcal{A}$ ➔ $\lambda\mathcal{A}$
which leaves the reduced shear *g* invariant under the transformation:

$$(\kappa \rightarrow 1 - \lambda + \lambda\kappa)$$



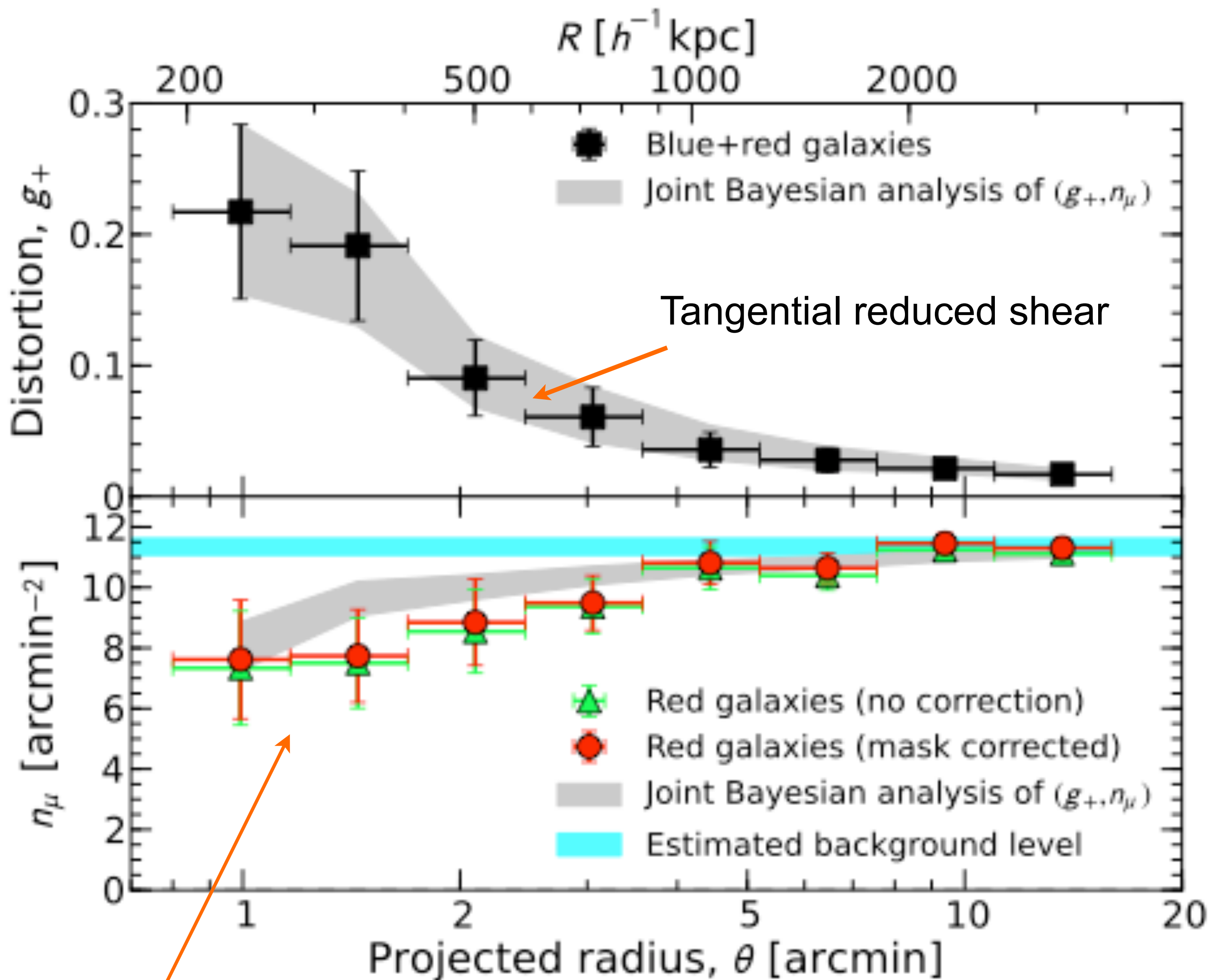
- can be removed by measuring independently the magnification, since "*magnification bias*", or *number counts depletion* : $\mu \propto \lambda^{-2}$

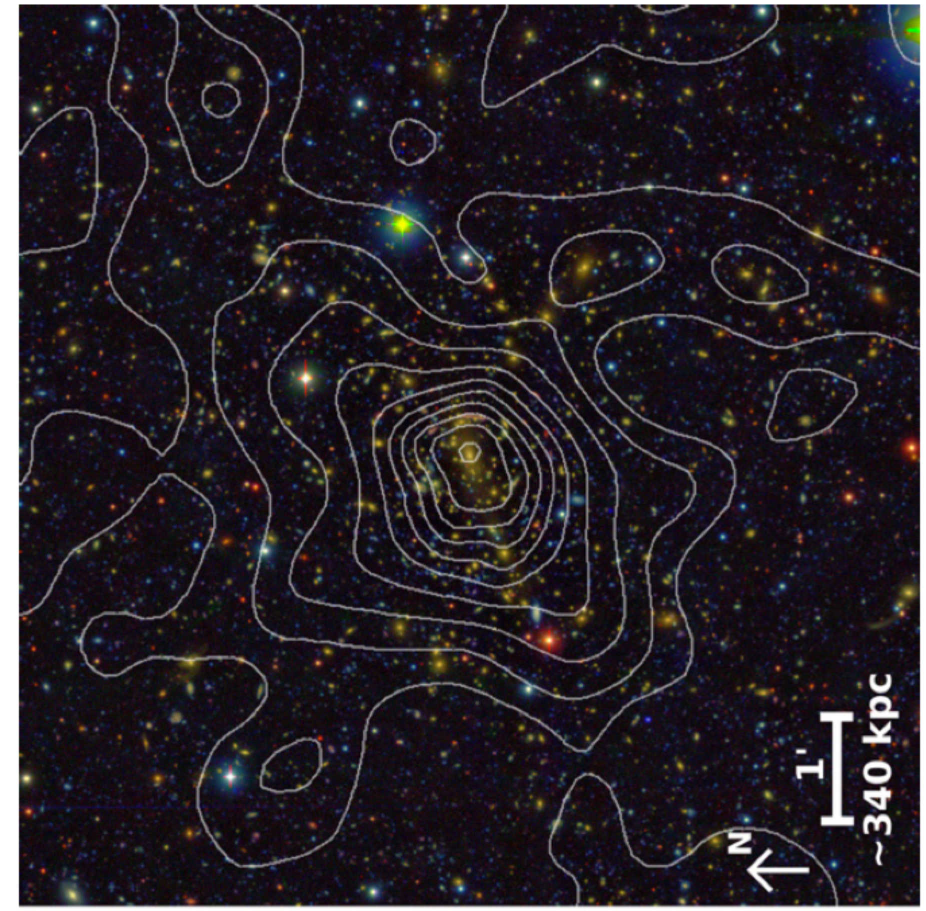
$$N'(m) = N_0(m)\,\mu^{2.5\,s-1} \qquad s = \frac{d\log N(m)}{dm}$$

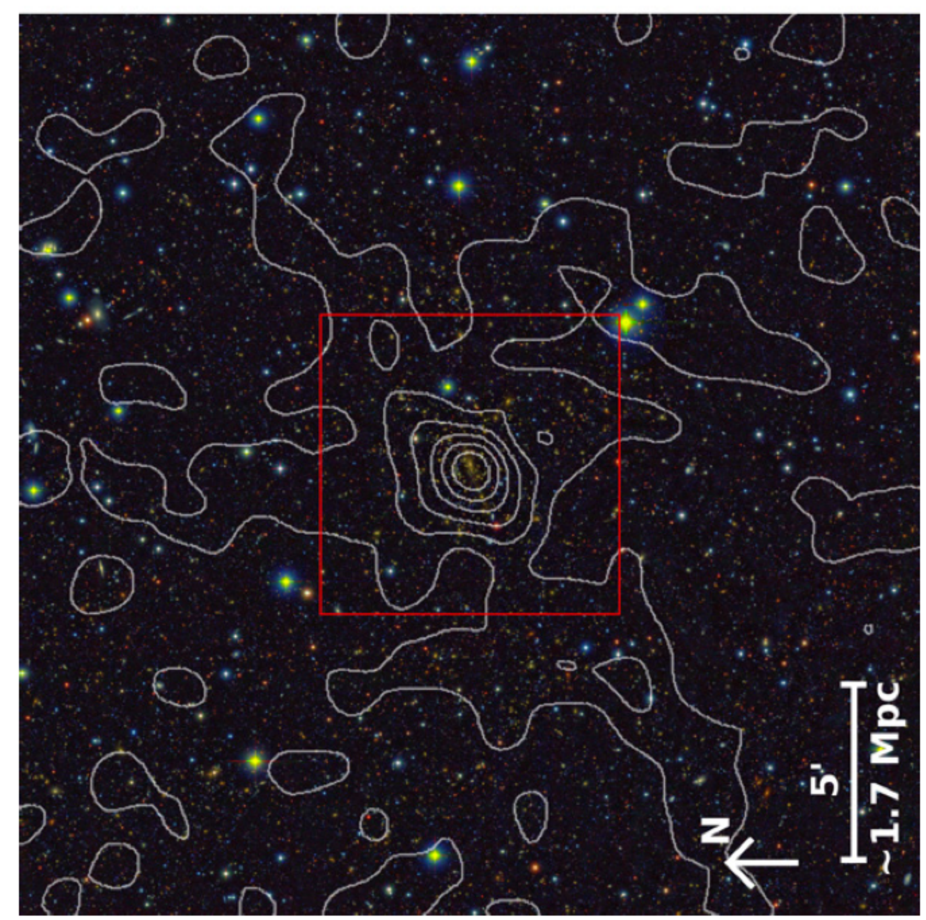
(Broadhurst et al. 95)

# Weak Lensing Analysis of MACS1206 Subaru imaging

(Umetsu et al. 2012)



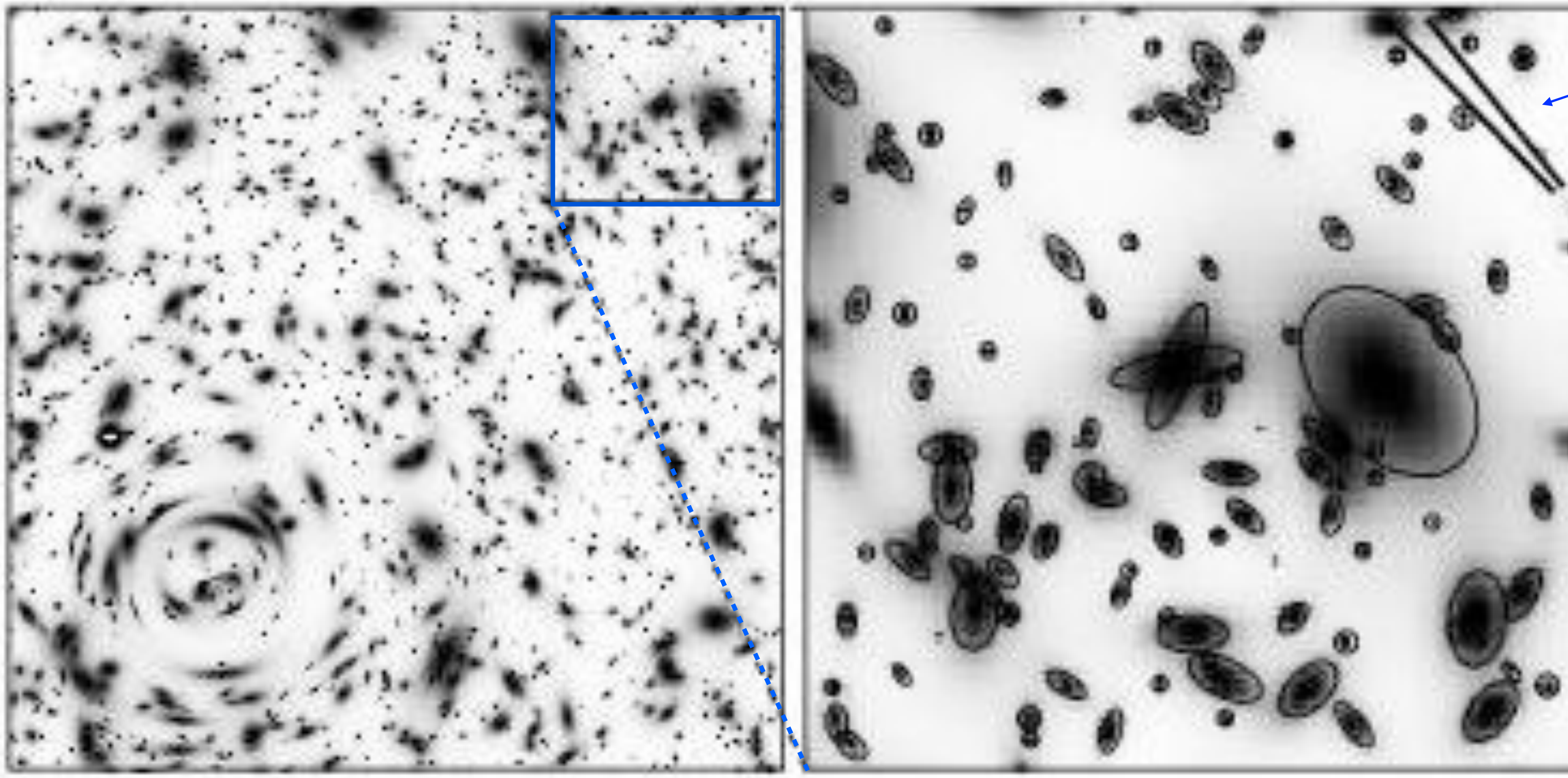




Number counts depletion
("magnification bias")
offers a model-independent way
of removing the
mass-sheet degeneracy

Mass contours

# Strong and Weak lensing from a cluster with projected surface mass density K(θ)




Mellier 2001

$K(\theta) = \Sigma(\theta)/\Sigma_{cr}$

$$\Sigma_{\rm cr} = \frac{c^2\, D_{\rm s}}{4\pi G\, D_{\rm d}\, D_{\rm ds}}$$

**Strong lensing regime**: K(θ) ≳ 1

Giant arcs, multiple images.
Parametric and non-parametric techniques to invert the lensing equation,

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

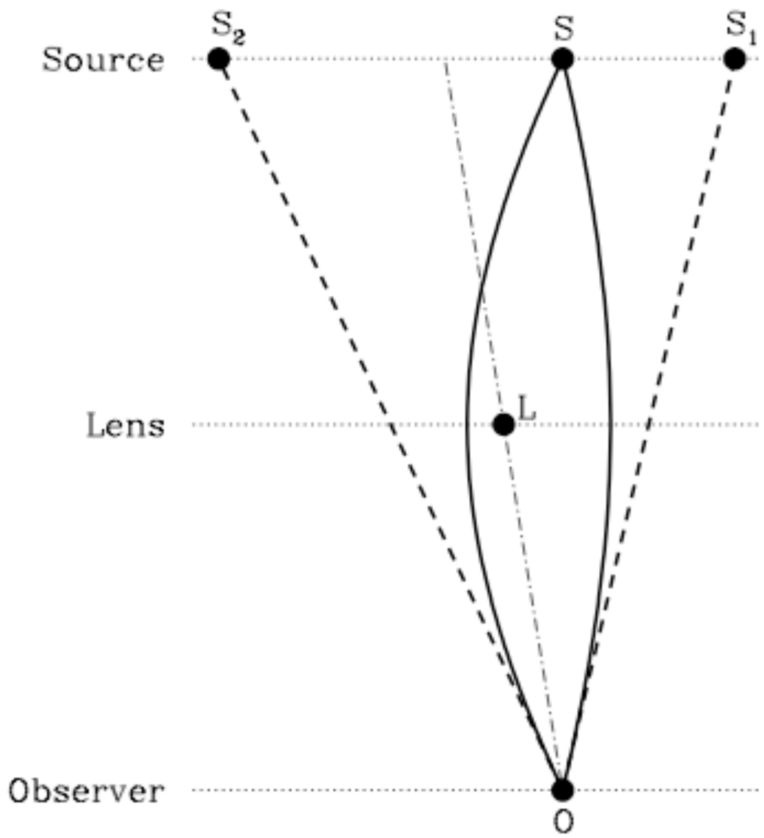
thus determining the deflection field and hence Σ(θ):

$$\vec{\alpha}(\vec{\theta}) = \vec{\nabla}\psi = \frac{1}{\pi}\int \kappa(\vec{\theta}')\,\frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2}\,d^2\theta'$$

**Weak lensing regime**: K(θ) << 1

From the statistical distortion of background galaxy shapes (averaged ellipticities) ➔ PSF corrected reduced shear ➔ K(θ) ➔ if the redshift distribution of the background galaxies is know the mass distribution Σ(θ) can be inverted up to a constant

# Time delay and the Hubble constant




$$(\vec{\theta}-\vec{\beta})-\vec{\nabla}_\theta\psi=0,$$

or

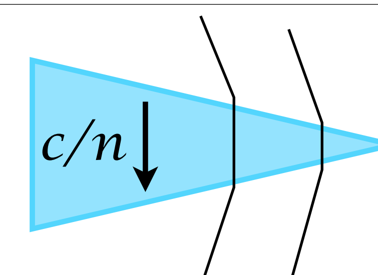
$$\vec{\nabla}_\theta\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2-\psi\right)=0.$$

lens equation

$$\vec{\nabla}^2\psi = 2\frac{\Sigma}{\Sigma_{cr}} \equiv 2\kappa$$

"Fermat" potential $\phi(\theta,\beta)$

$$\tau(\vec{\theta},\vec{\beta})=\tau_{\rm geom}+\tau_{\rm grav}=\frac{1+z_{\rm L}}{c}\frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}\left(\frac{1}{2}(\vec{\theta}-\vec{\beta})^2-\psi(\theta)\right).$$

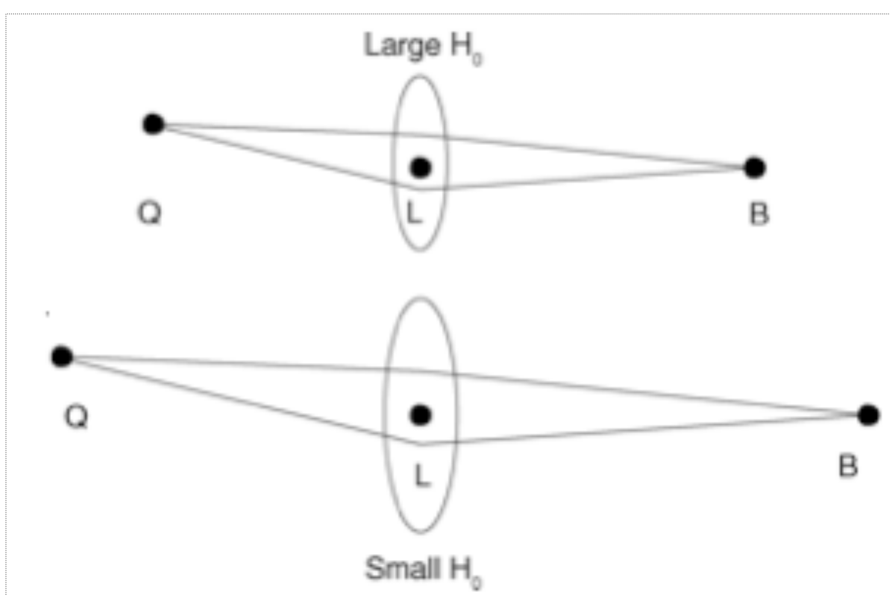



Masses bend passing light similarly to convex lenses.

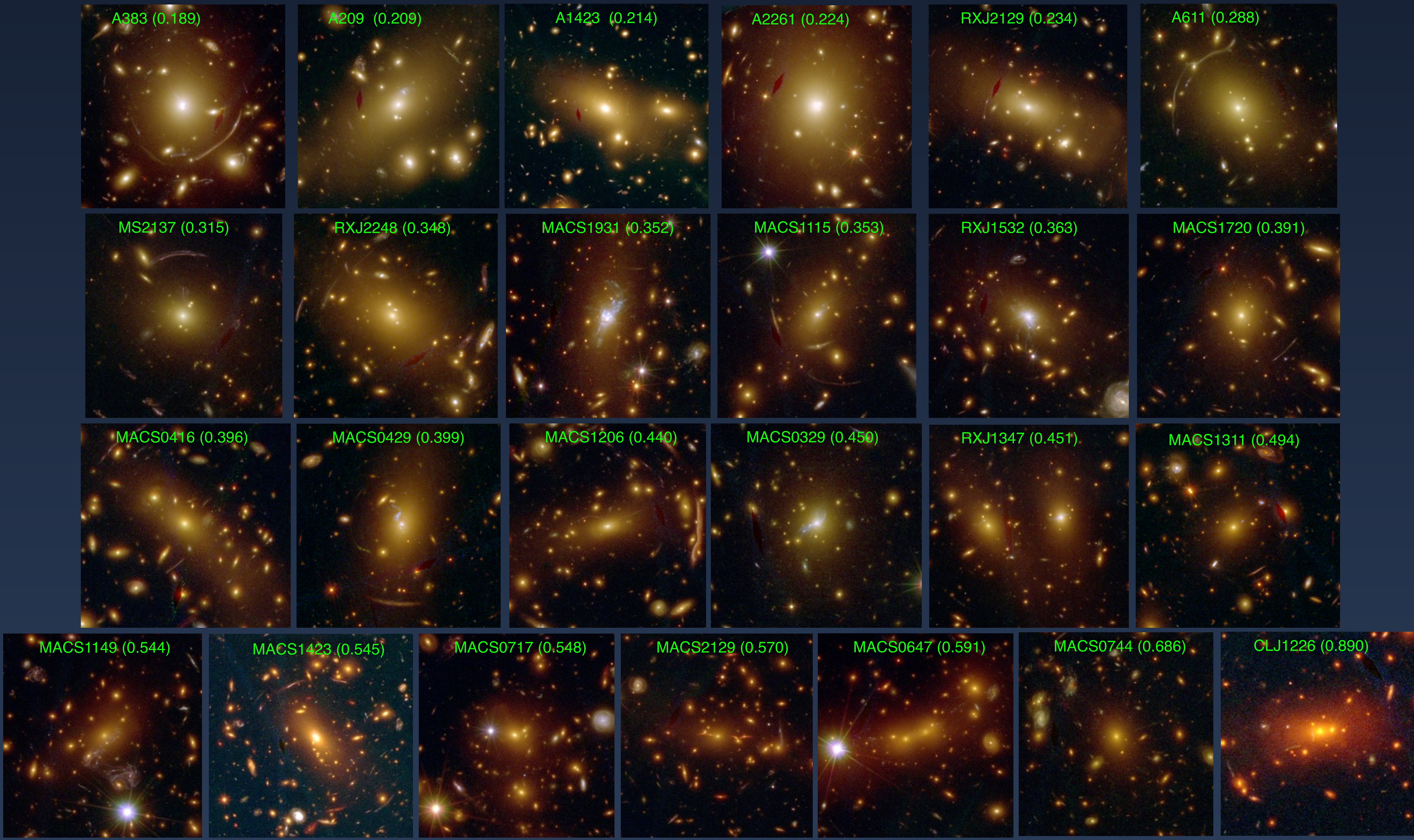
Fermat's principle in gravitational lensing optics for a medium with an index of refraction $n=1-\frac{2\Phi}{c^2}>1$

Images occur where the τ is extremal, i.e. $\vec{\nabla}_\theta\tau=0.$

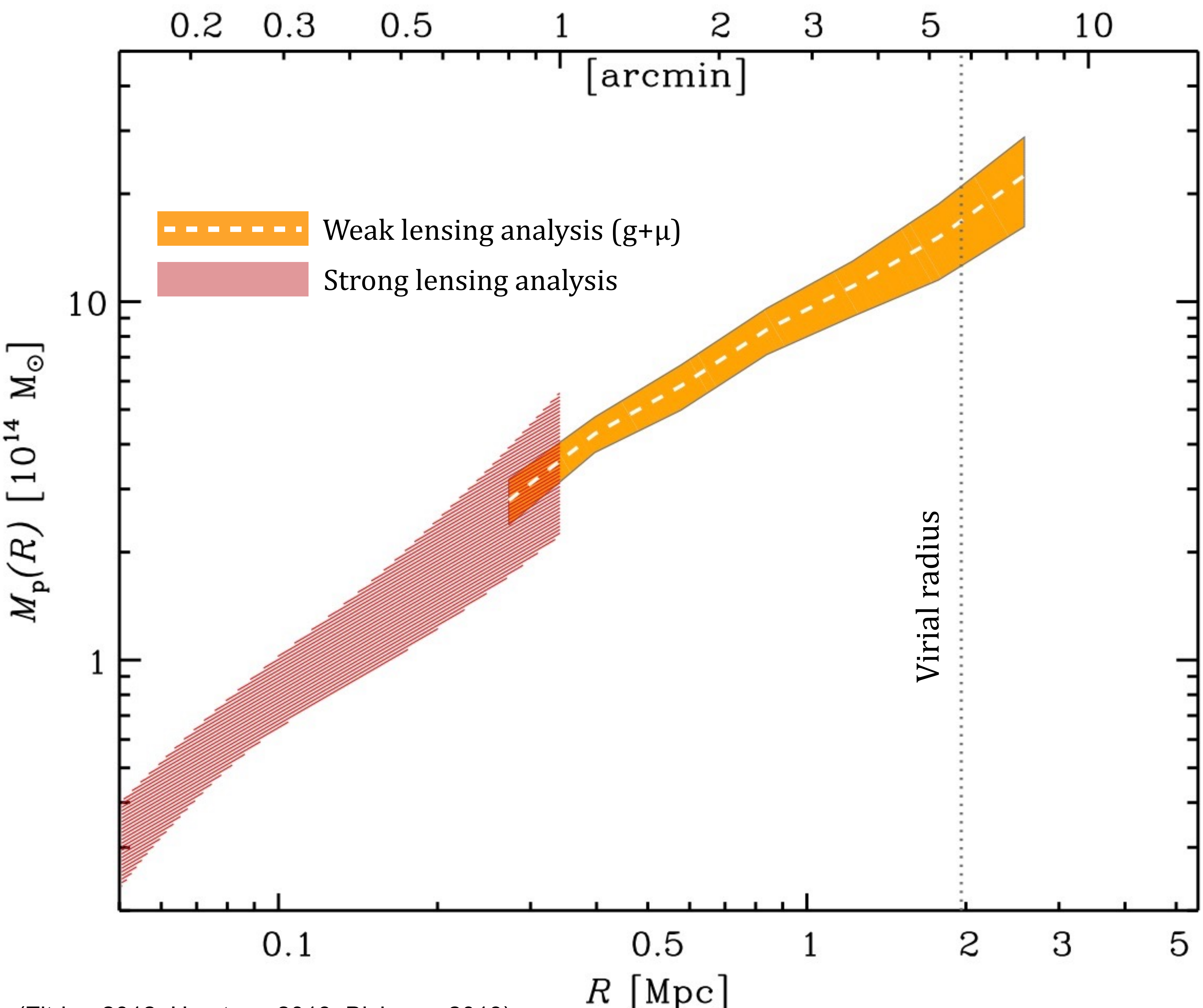
Time delay ~ $H_0^{-1}$ → if a robust model is available for the lensing potential, *ψ(ϑ)*, then by monitoring the time delay of variable sources (QSOs) $H_0$ can be measured in one step (Refsdal 1964).



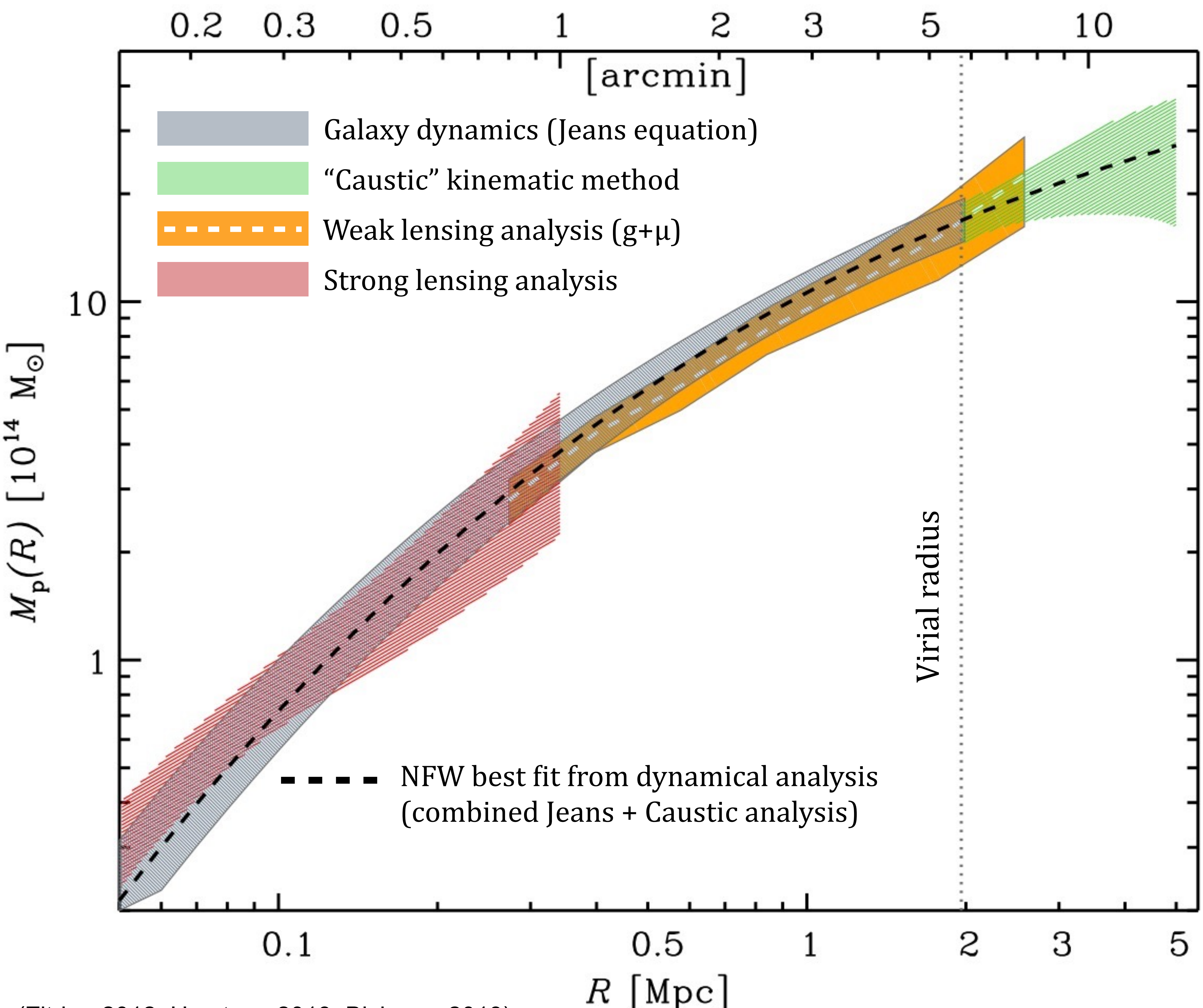
# CLASH Gallery: All 25 Clusters



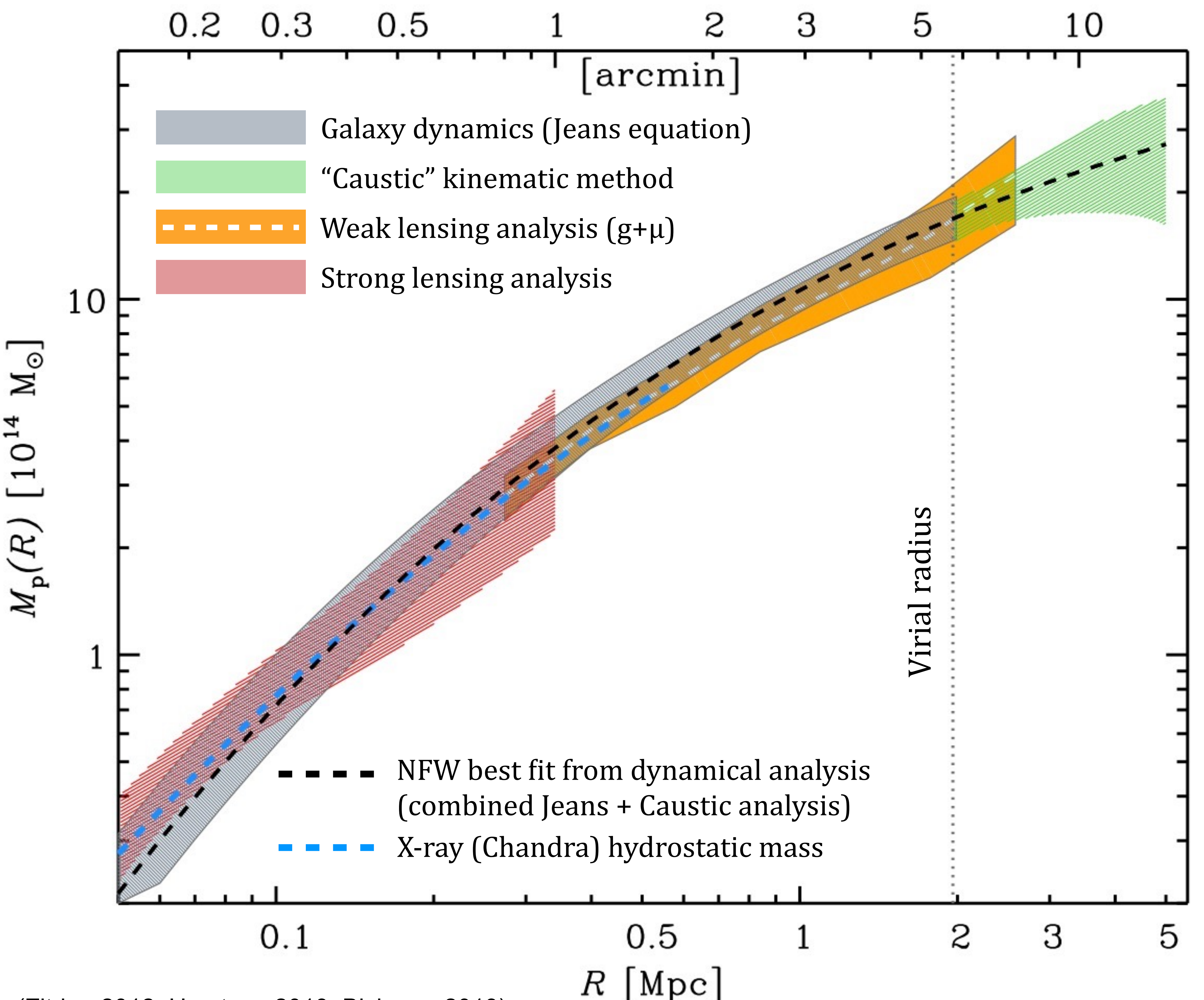

All HST observations completed in July 2013. Data products in the STScI Archive.




(Zitrin+ 2012, Umetsu+ 2013, Biviano+ 2013)




(Zitrin+ 2012, Umetsu+ 2013, Biviano+ 2013)

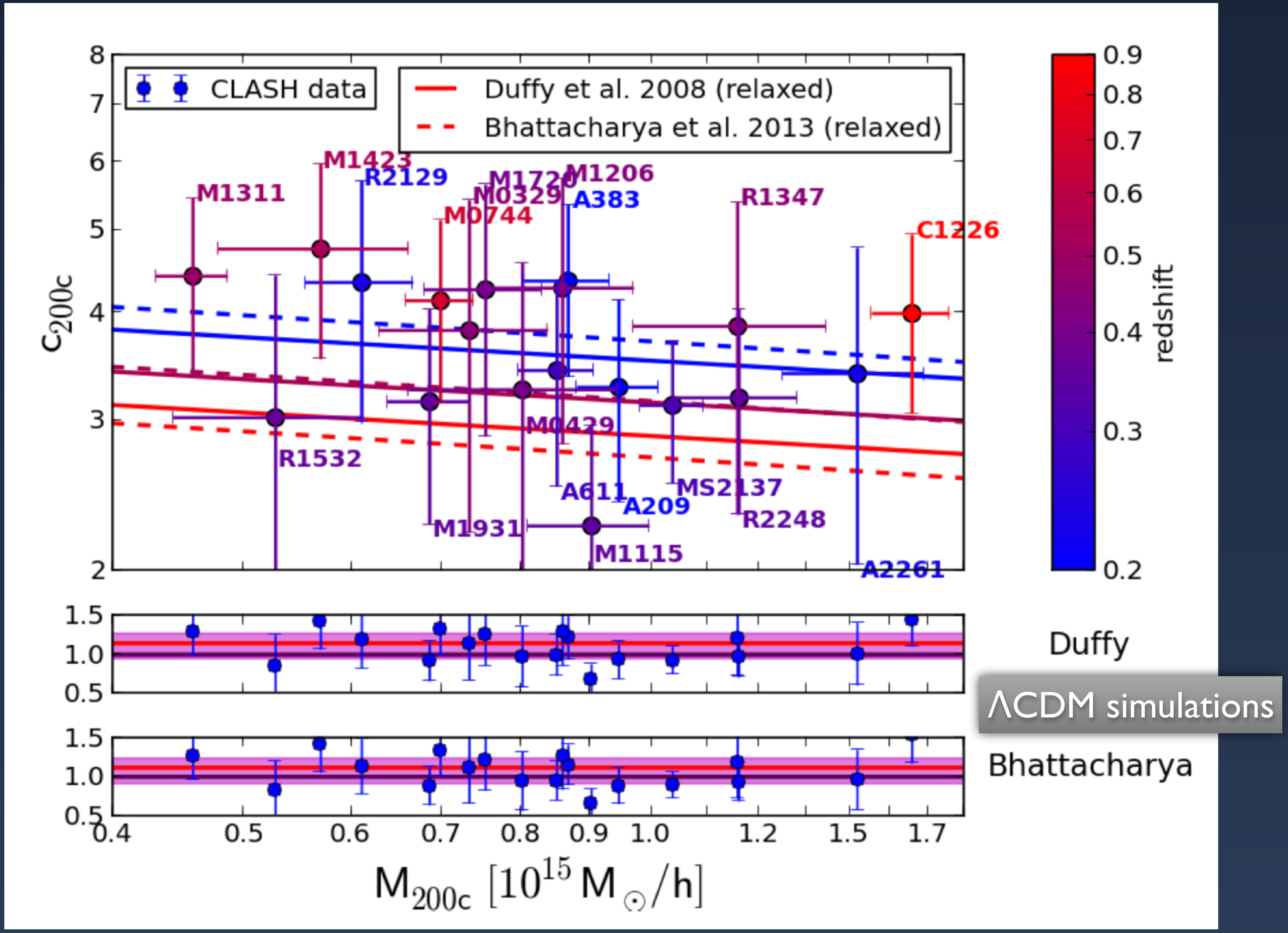



(Zitrin+ 2012, Umetsu+ 2013, Biviano+ 2013)

# Concentration – Total Mass Relationship

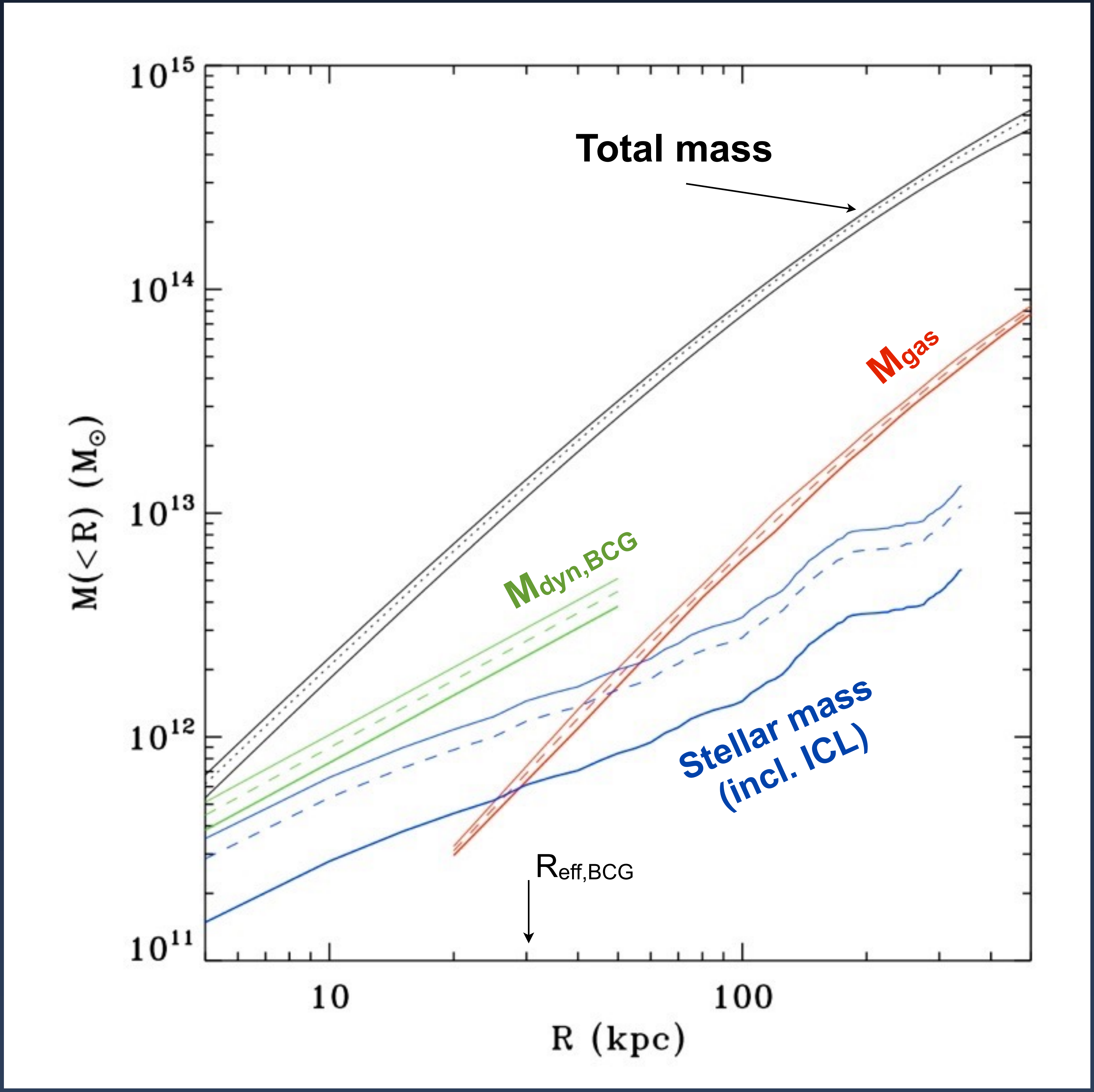
(J.Merten et al. ApJ, 2014)

NFW fits of weak & strong lensing profiles from 19 CLASH X-ray selected clusters

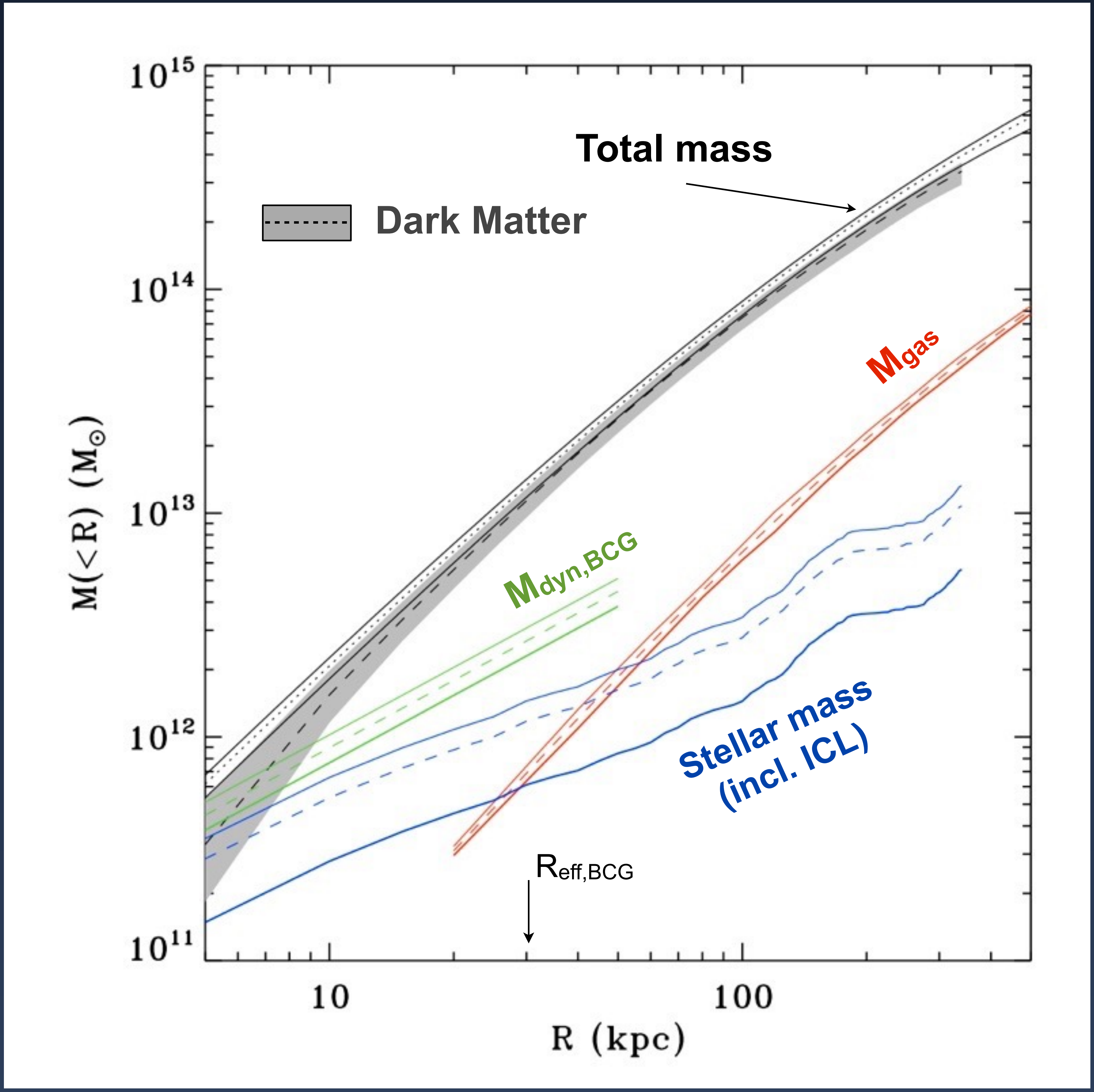



➔ No significant tension with predicted c-M relation in ΛCDM

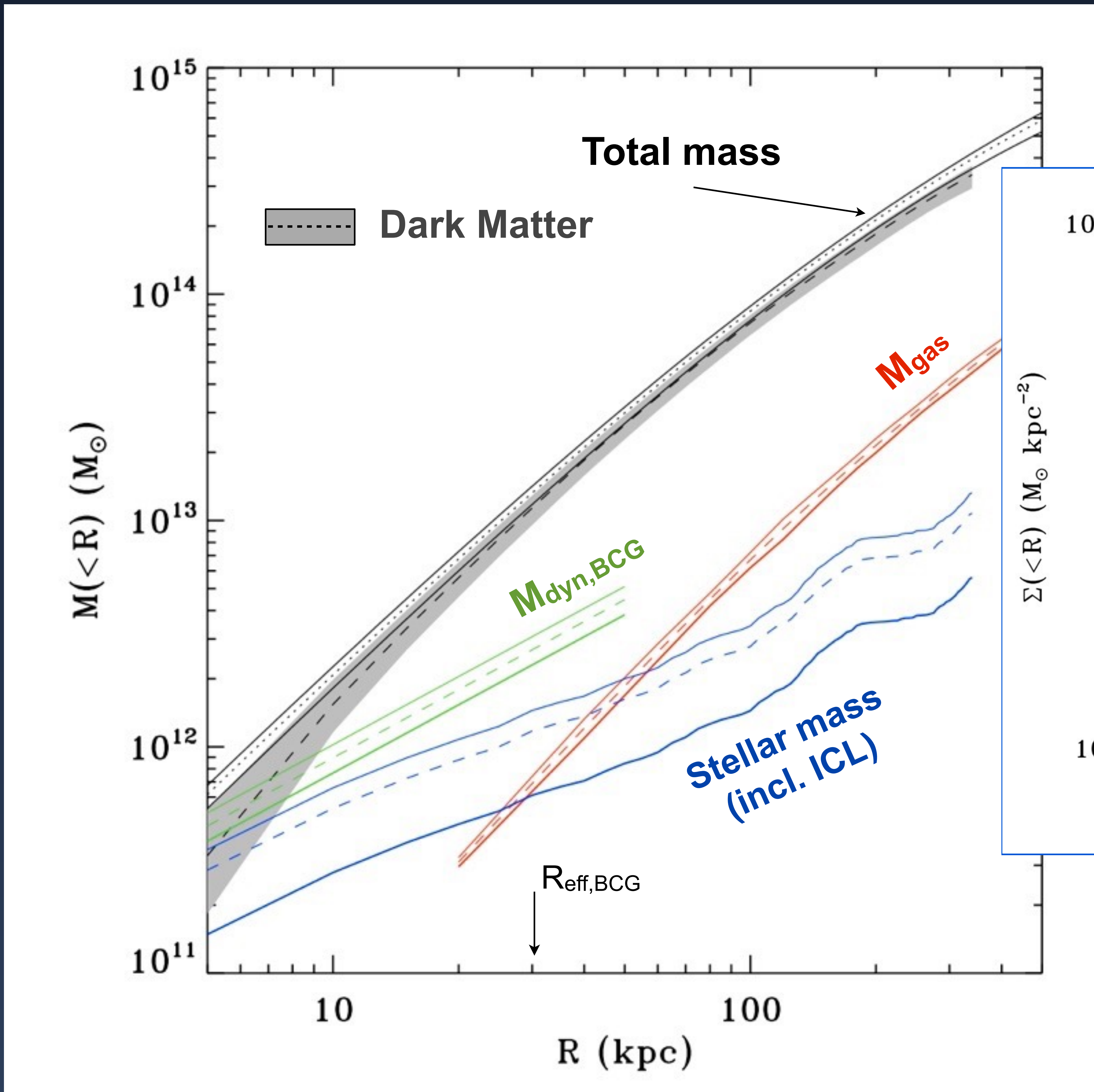
# Decomposing baryons and DM in the inner core of MACS1206

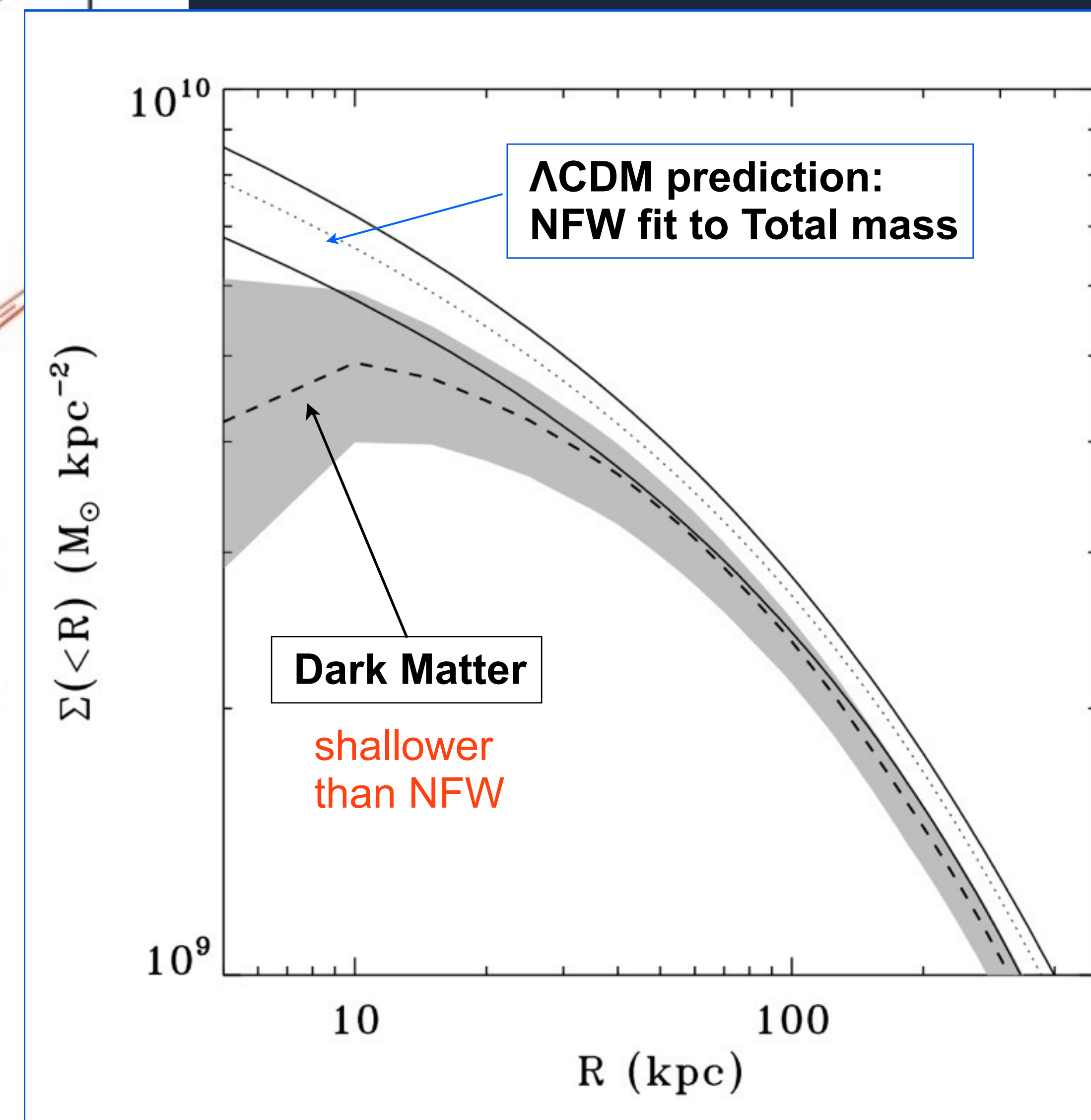


# Decomposing baryons and DM in the inner core of MACS1206



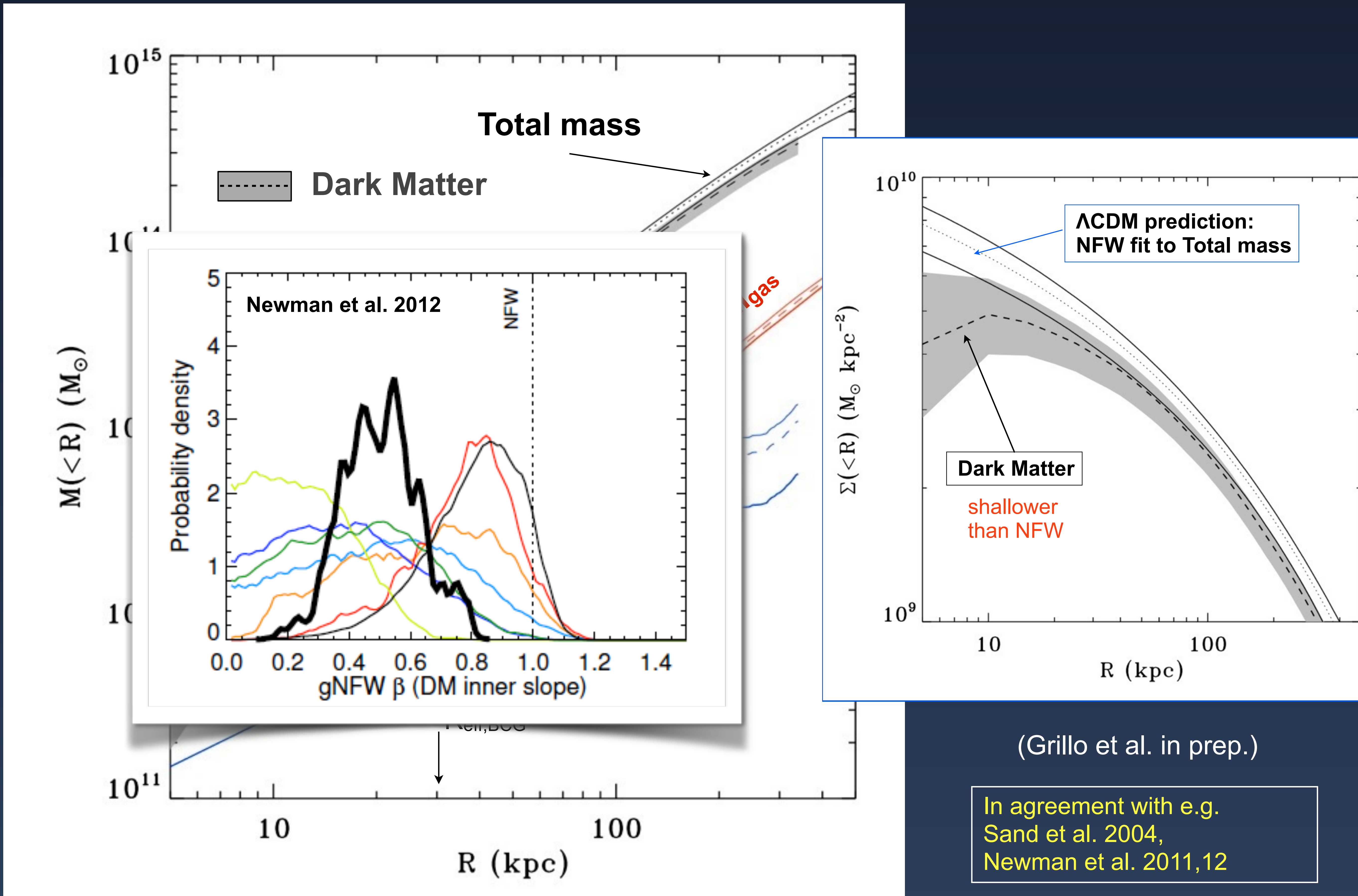
# Decomposing baryons and DM in the inner core of MACS1206




(Grillo et al. in prep.)

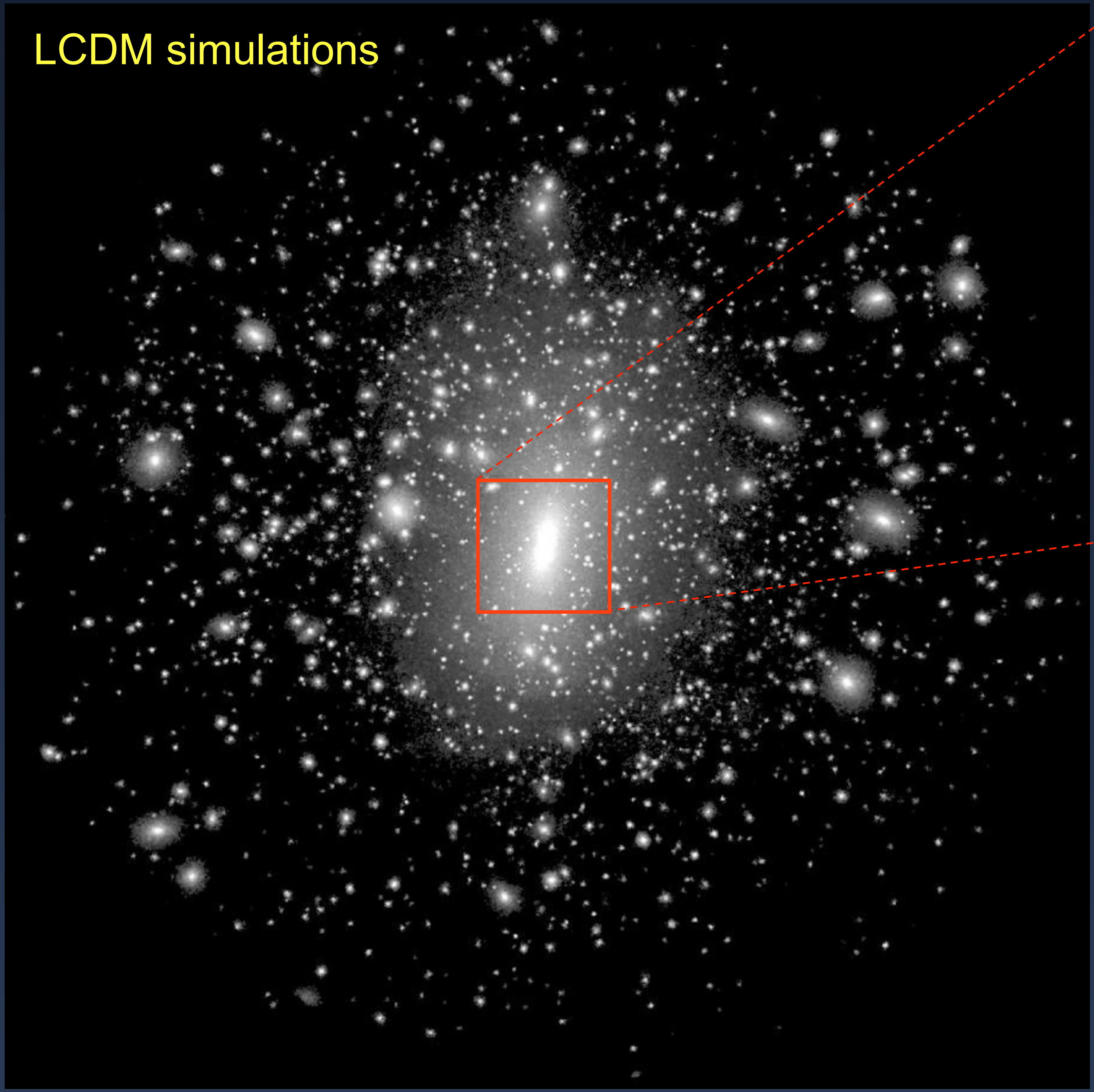
# Decomposing baryons and DM in the inner core of MACS1206




(Grillo et al. in prep.)

In agreement with e.g.
Sand et al. 2004,
Newman et al. 2011,12

# Strong lensing can resolve dark matter halos !

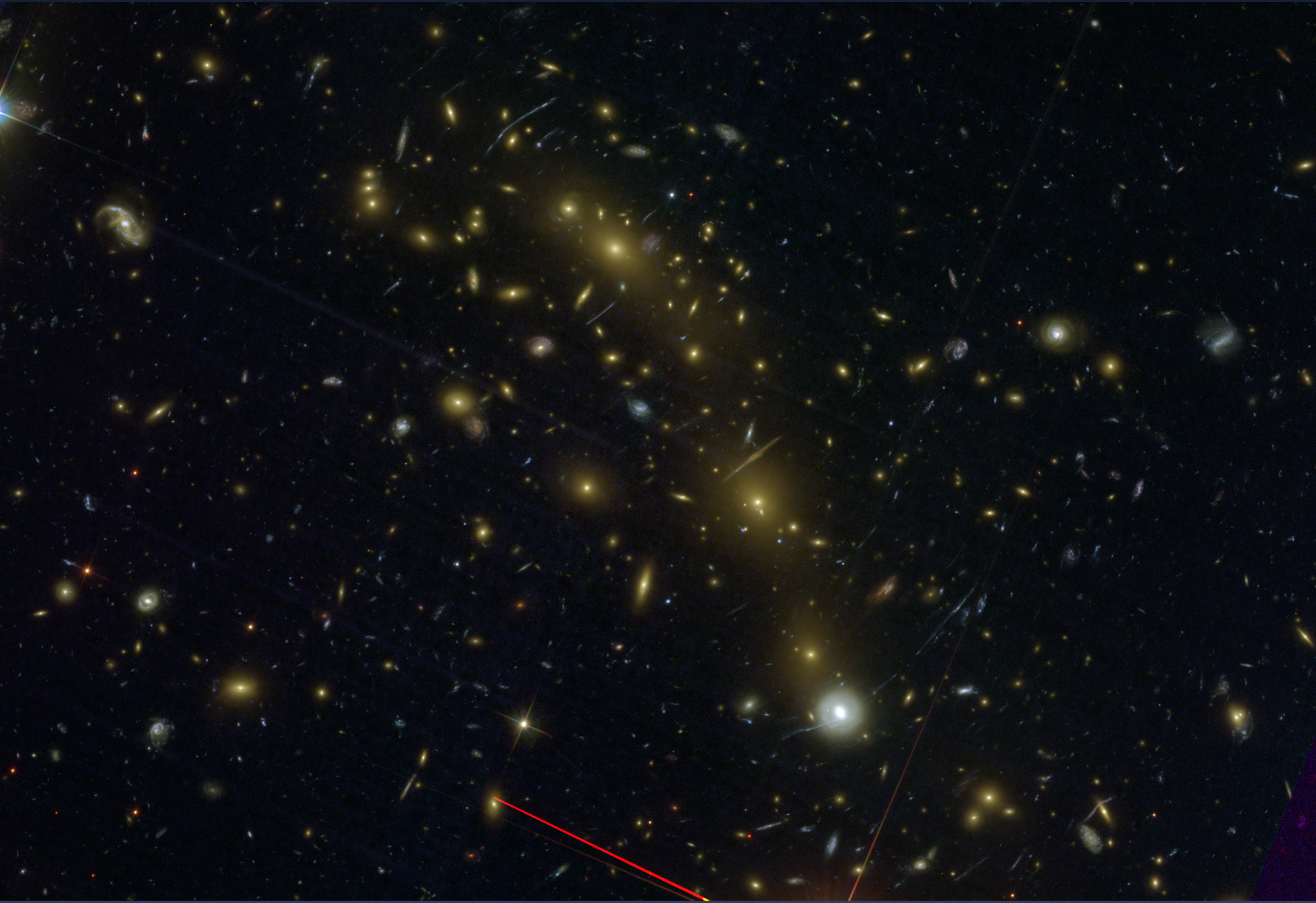



Dark matter density distribution from a high resolution simulation of a massive cluster to the virial radius (Diemand et al. 2005)

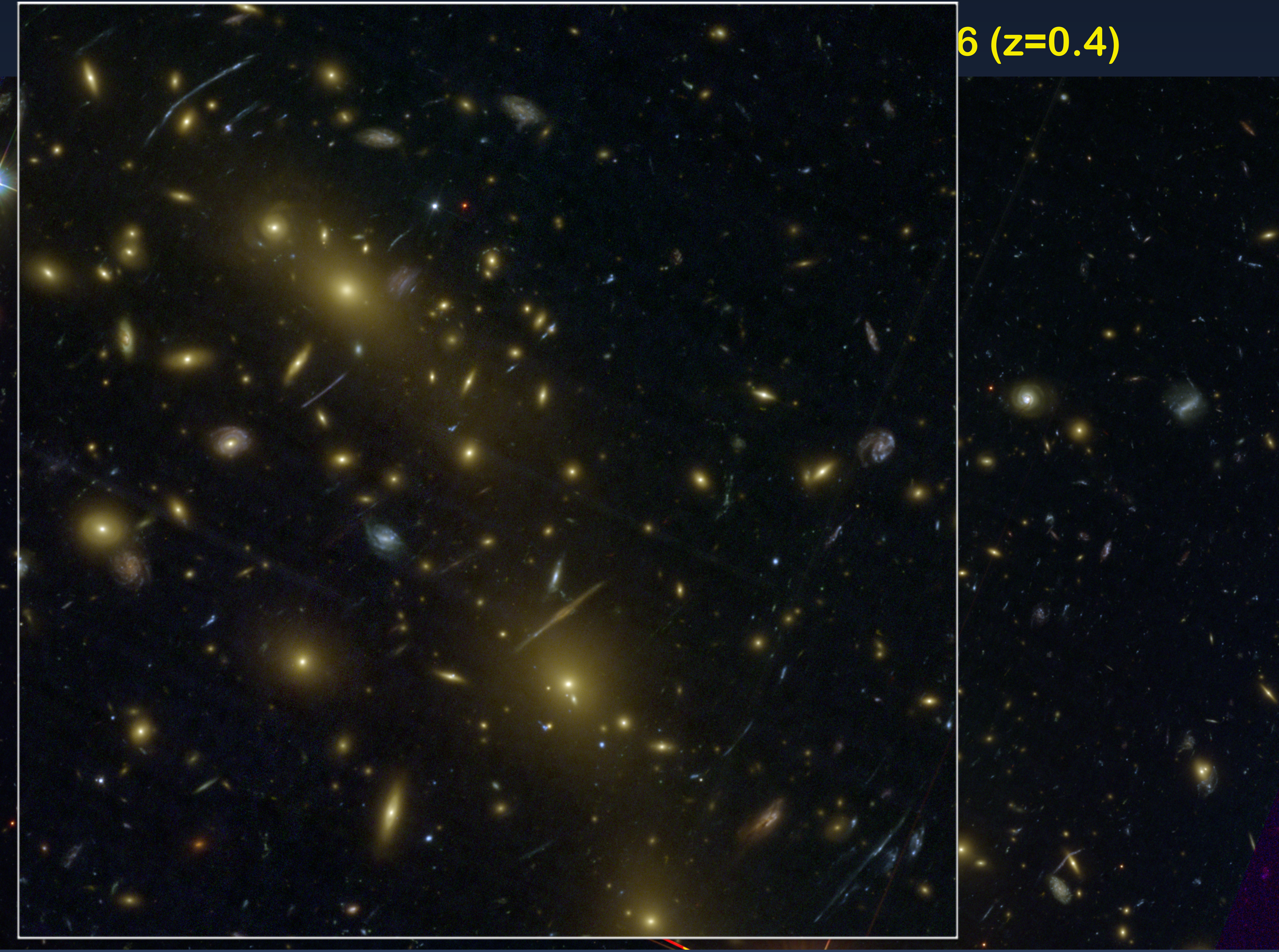

Deep HST image of massive cluster

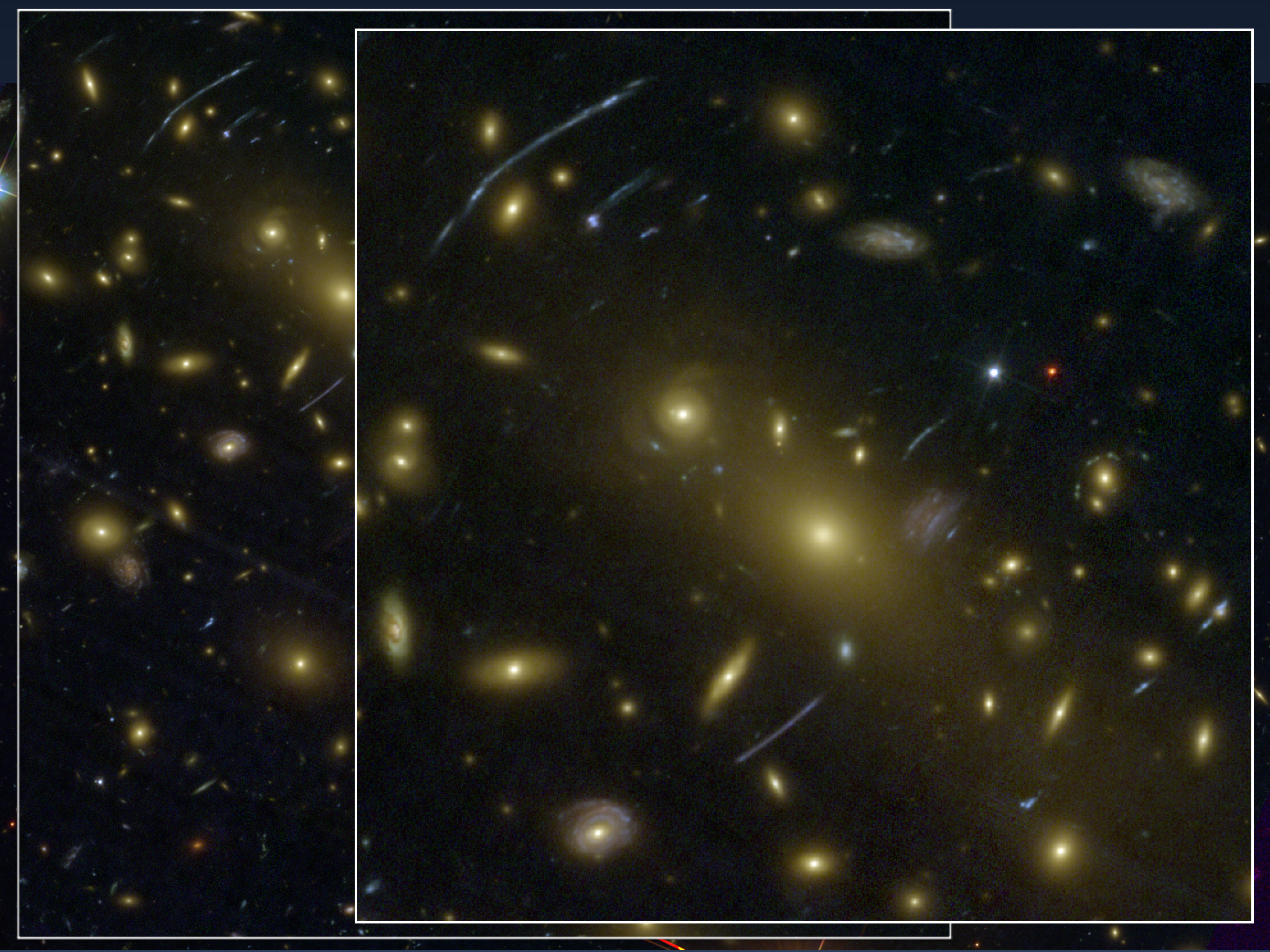
Reconstructed total mass with resolution $R \propto \frac{\theta_{\mathrm{Einstein}}}{\sqrt{N_{\mathrm{Arcs}}}}$

# Detailed DM halo structure of MACS0416 (z=0.4)

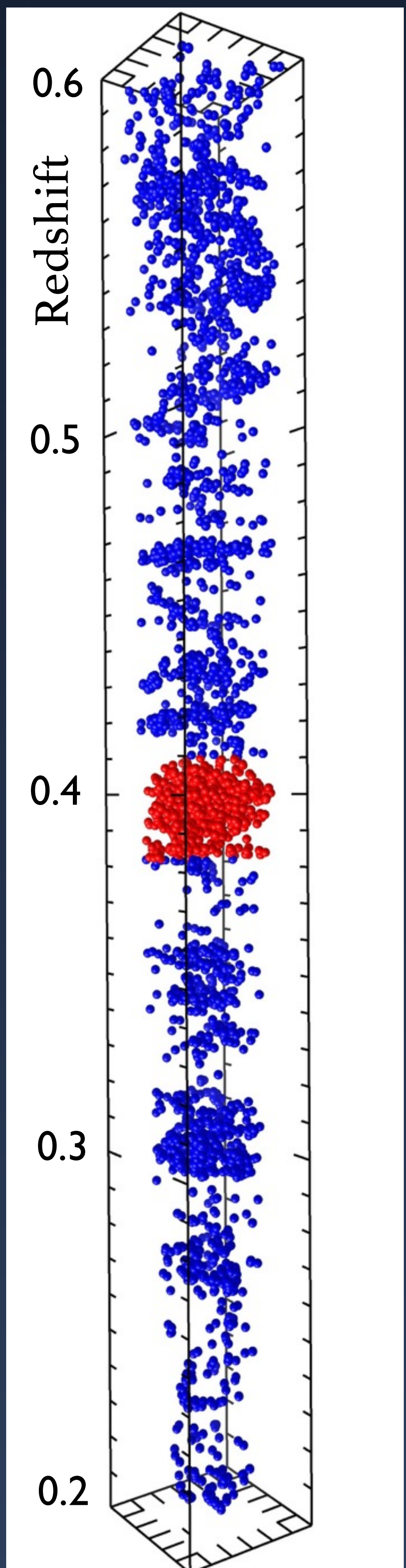


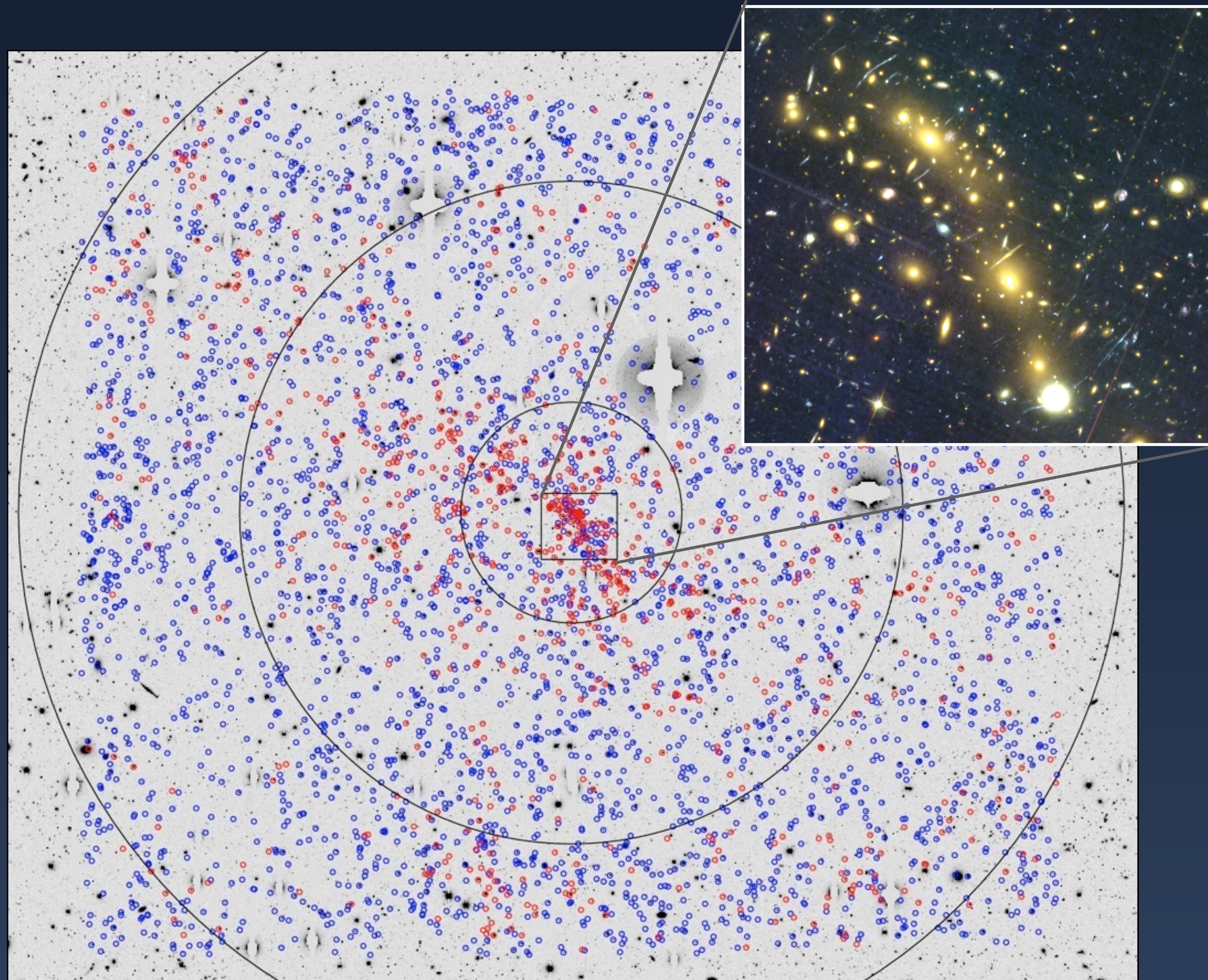
# 6 (z=0.4)



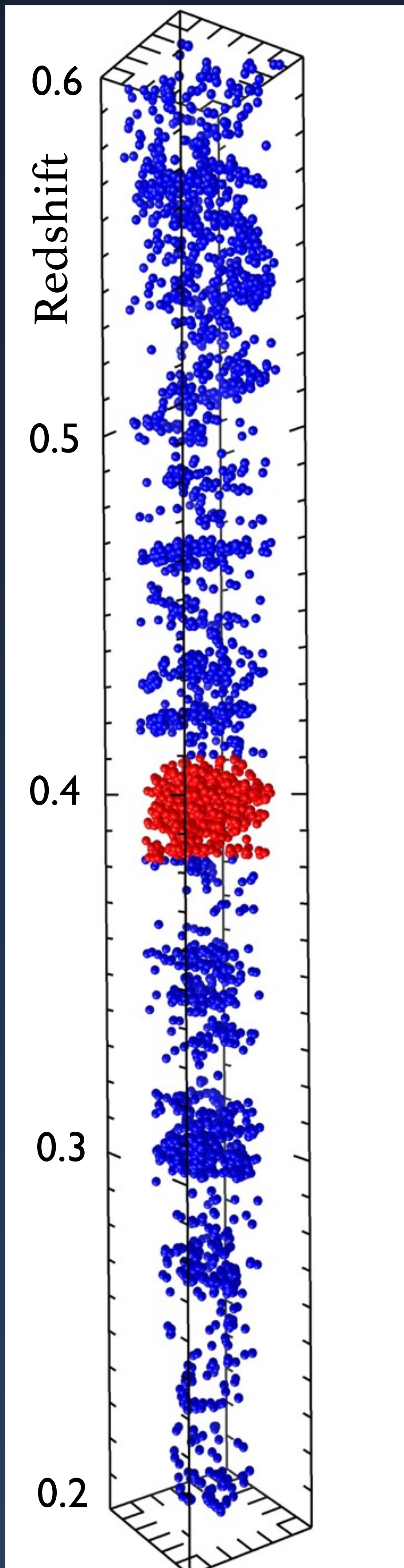


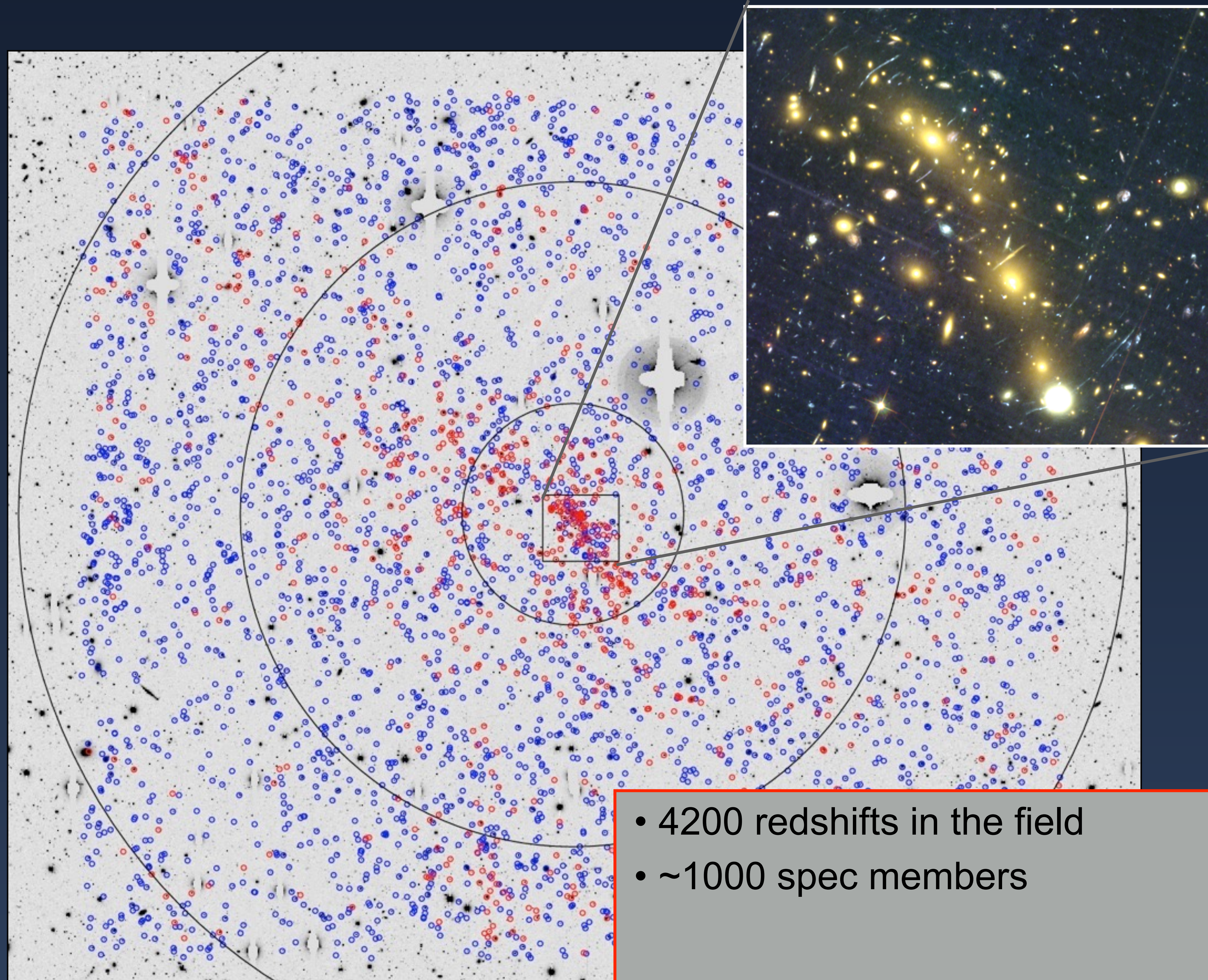
# CLASH-VLT spectroscopic campaign of MACS0416



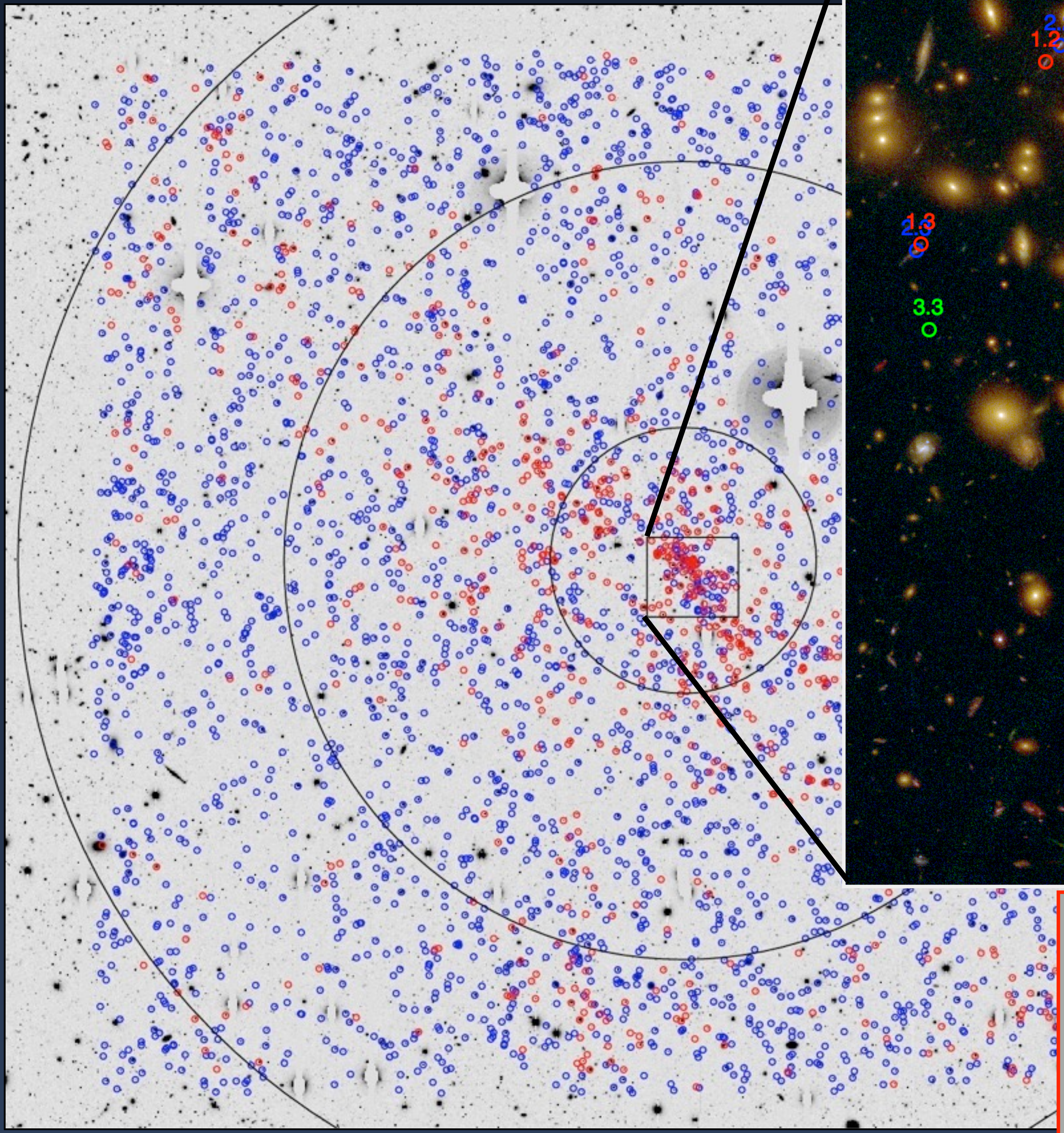
# CLASH-VLT spectroscopic campaign of MACS0416

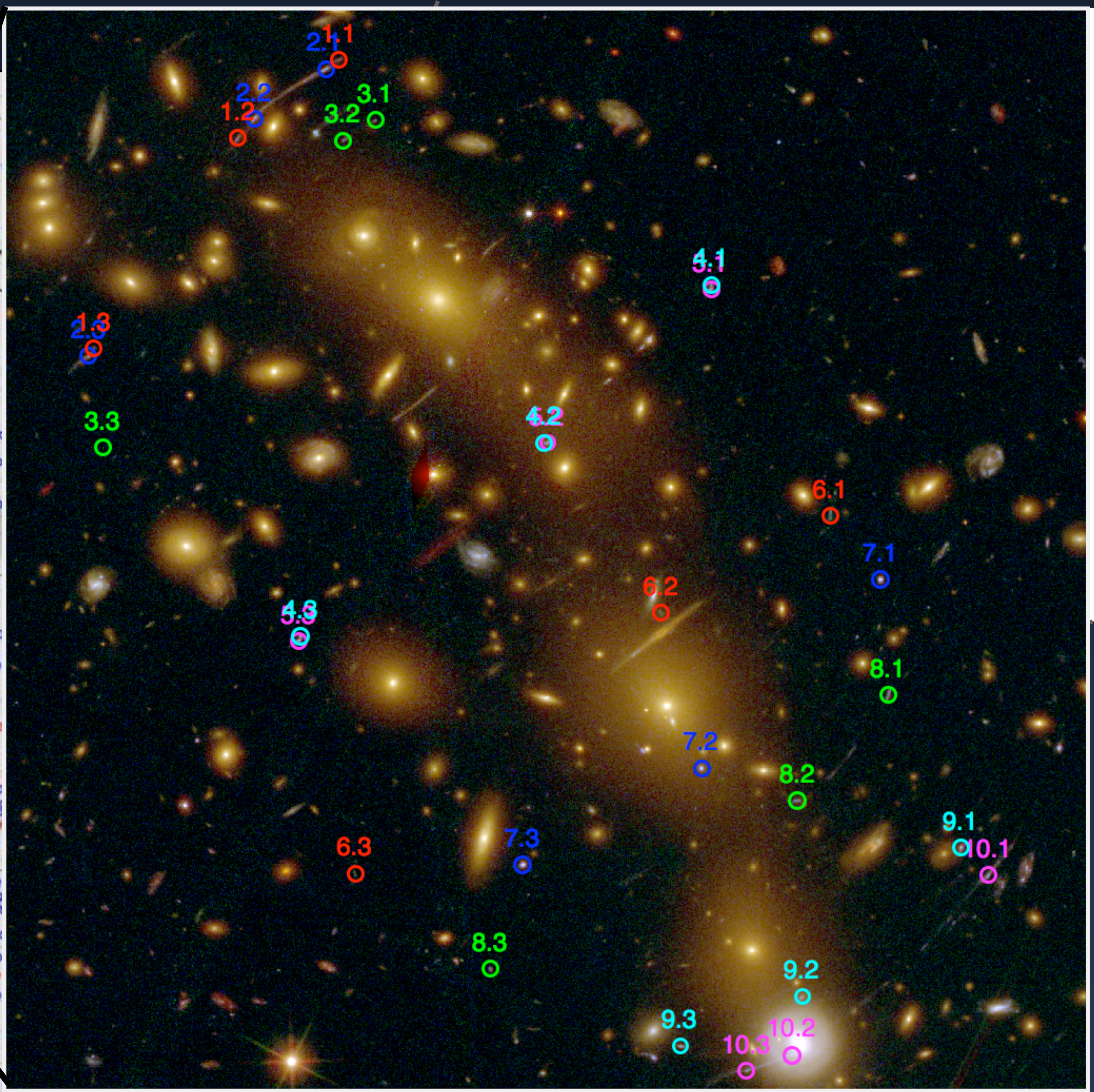


- 4200 redshifts in the field
- ~1000 spec members

# CLASH-VLT spectroscopic campaign of MACS0416



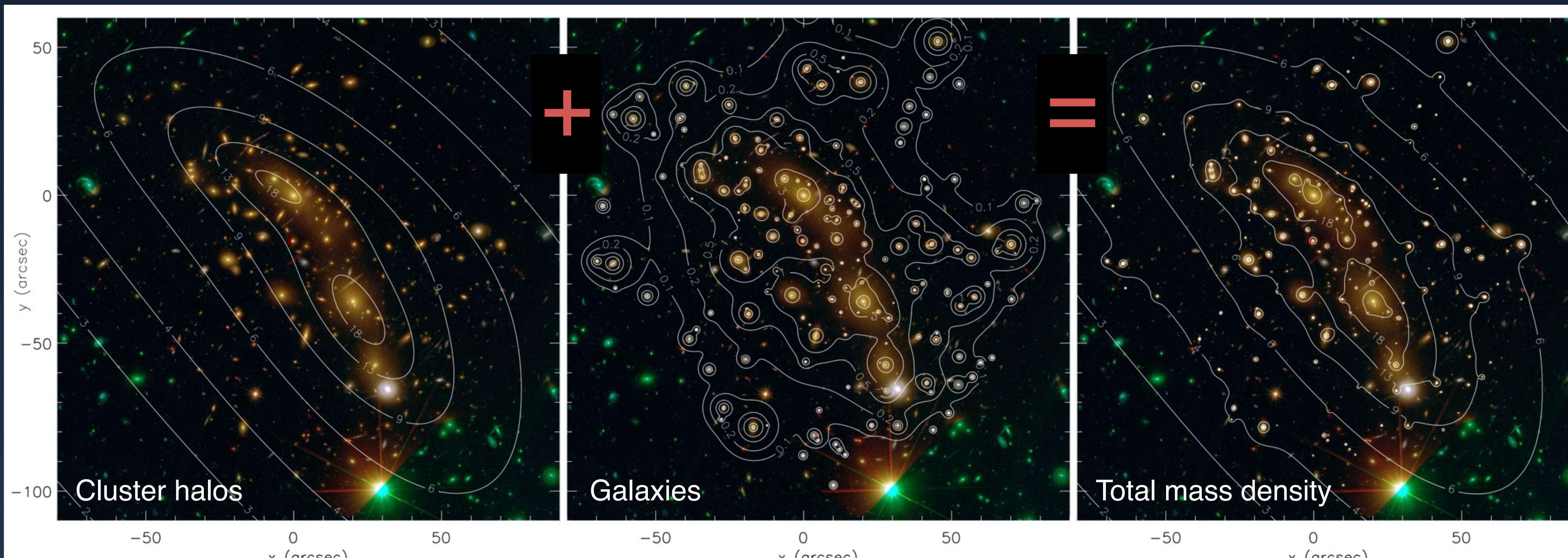



- 4200 redshifts in the field
- ~1000 spec members
- 10 multiple systems identified

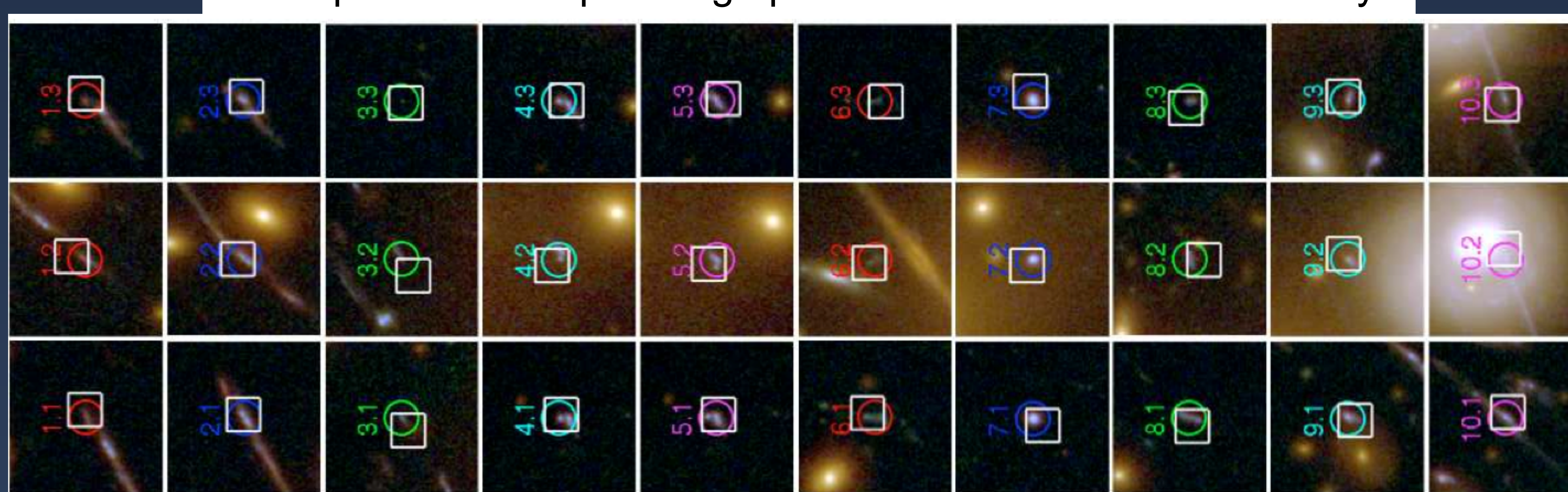
  30 multiple-images at 1.6<z<3.2

# Detailed DM halo structure of MACS0416

(Grillo et al.  2014)

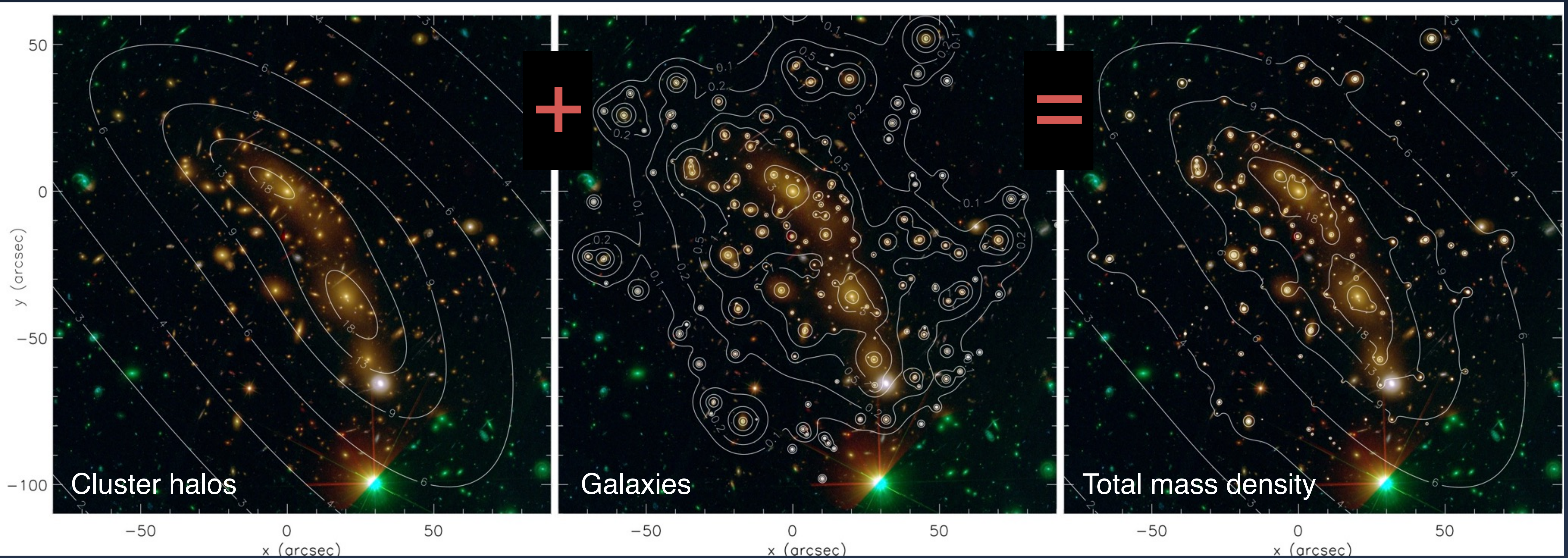



reproduce multiple image positions with ~0.3” rms accuracy



# Detailed DM halo structure of MACS0416

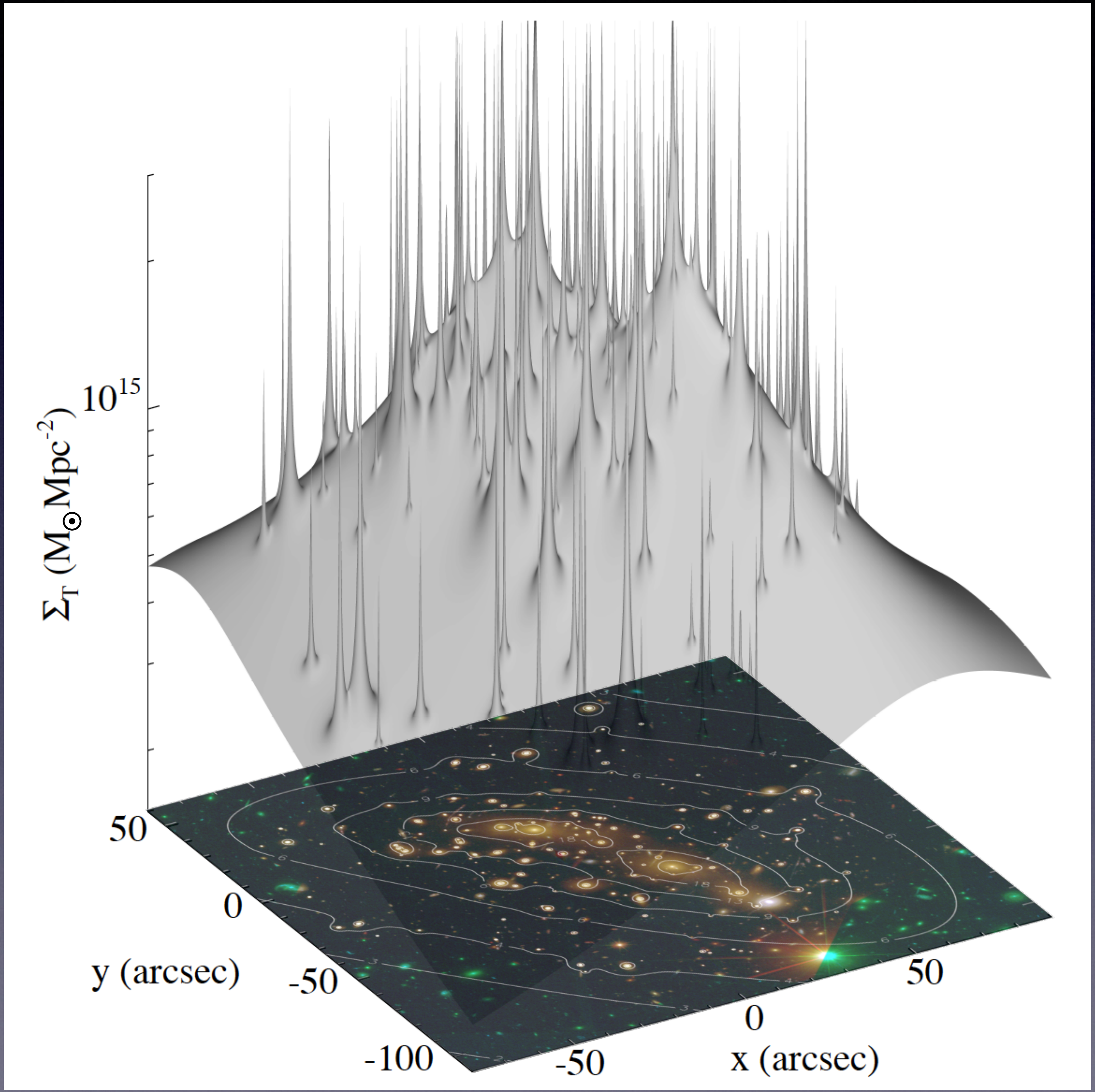
(Grillo et al.  2014)




Spectroscopic information of cluster and lensed galaxies is critical for accurate DM maps

# Resolving cluster mass distribution with strong lensing

(Grillo et al. astro-ph 1407.7866)

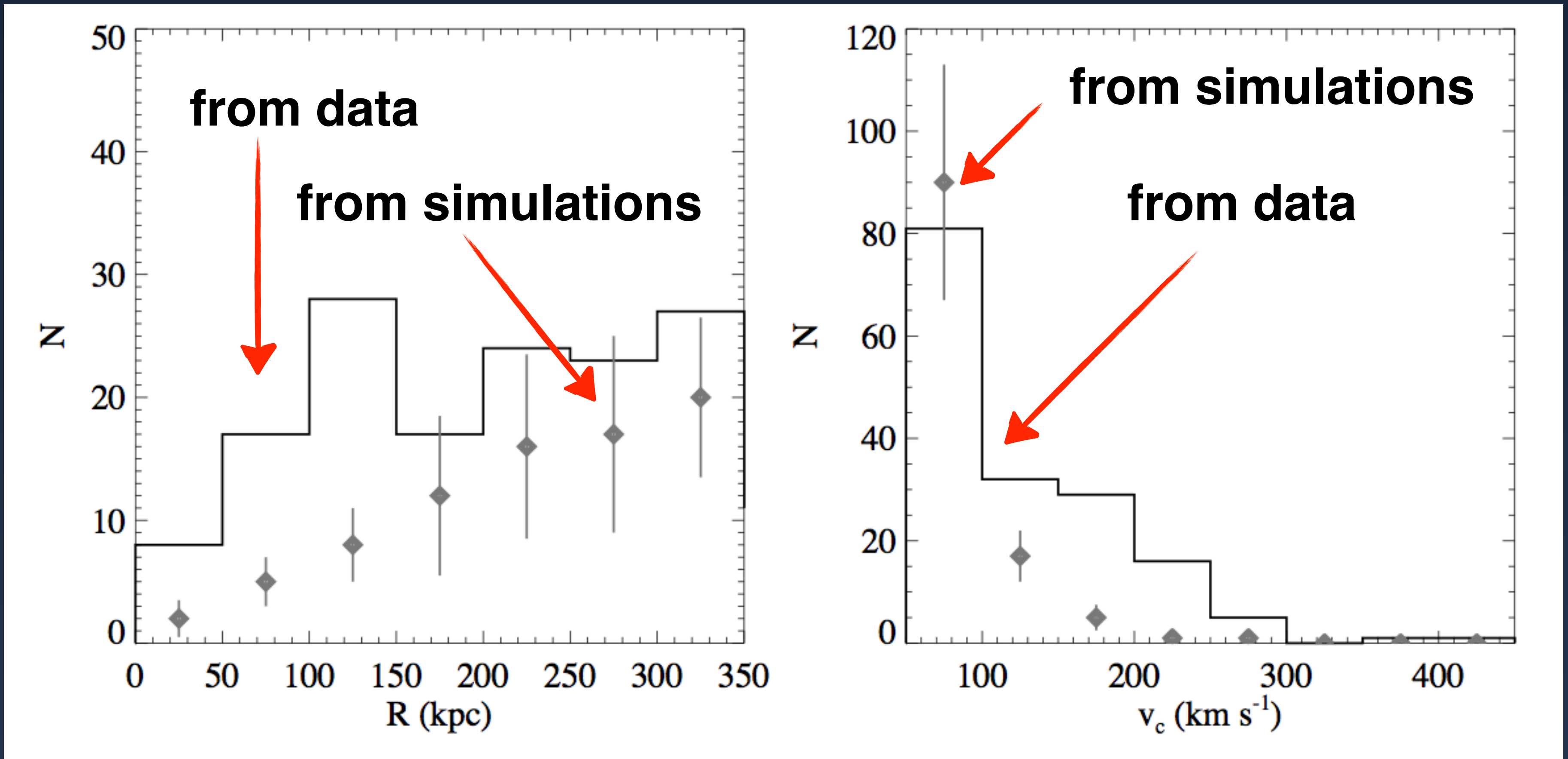


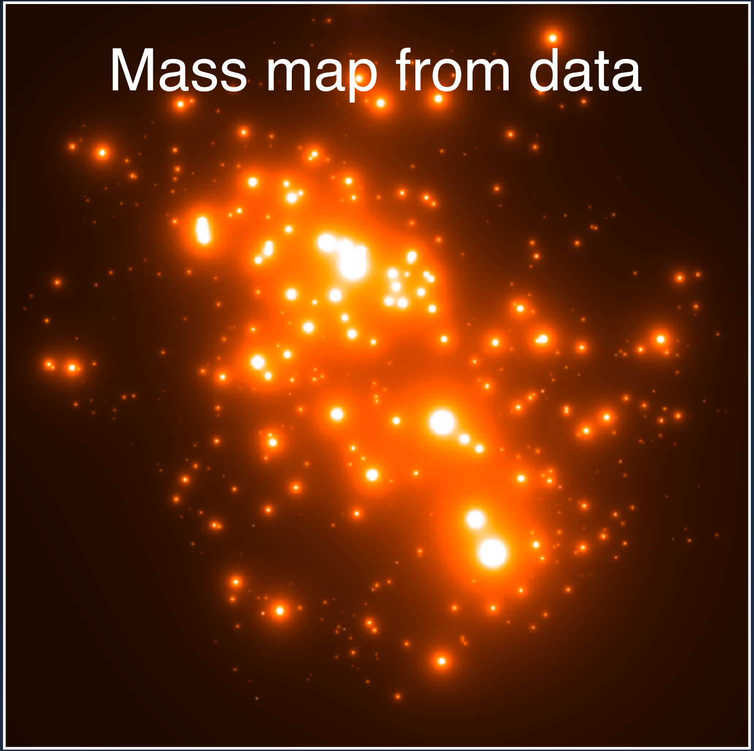
# DM halo structure: mass function of sub-halos

## Comparing with theoretical expectations

(Grillo et al. astro-ph 1407.7866)

### Distribution of sub-halos: observations vs simulations




24 simulated clusters
with similar masses
(S.Borgani's group)

First results indicate:

there is a lack of massive sub-halos in N-body DM only simulations, mostly located in the central regions

- tidal stripping of massive sub-halos ?
- what's wrong with the DM only simulations ?

# Constraining the DM Equation of State

(Sartoris et al. 2014)

Testing whether DM is pressureless p=0 (method proposed by Faber&Visser 2006)

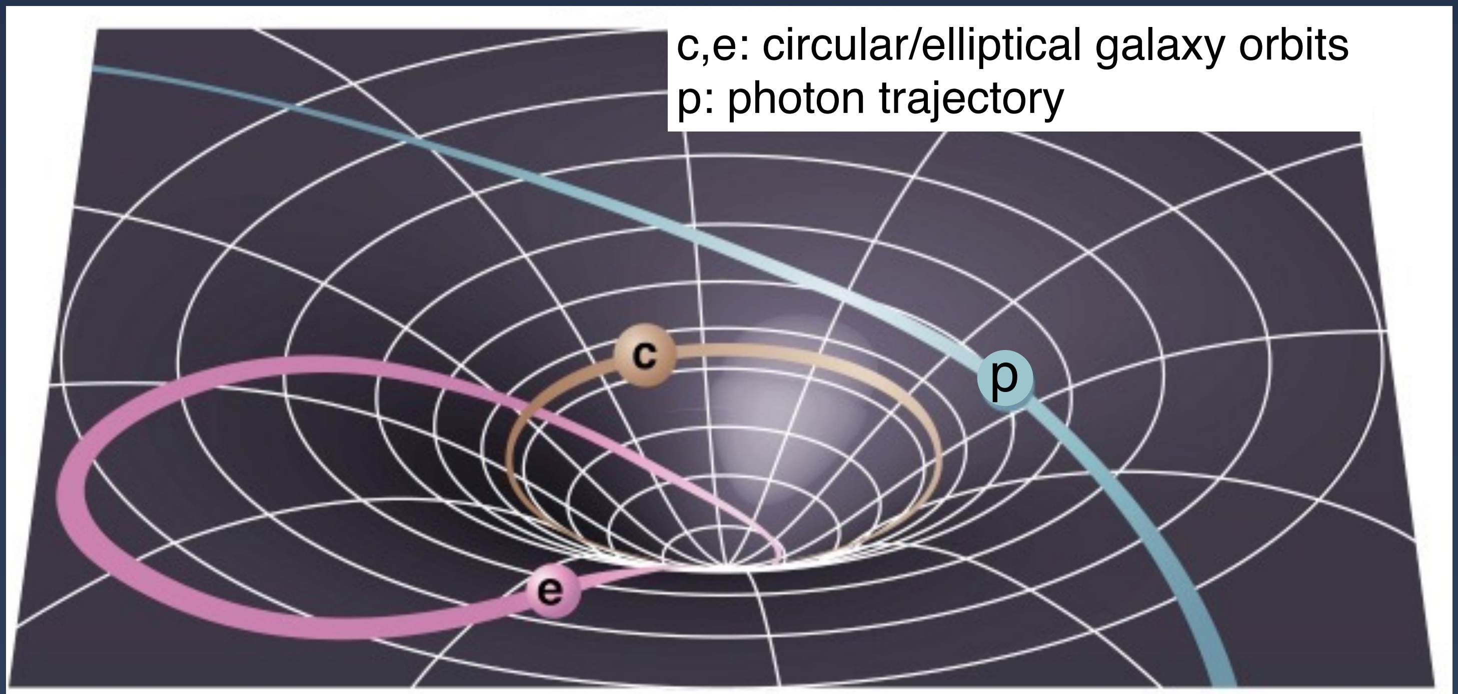
Made possible by our high-quality lensing and kinematic mass profiles for MACS1206, a "well relaxed cluster" with negligible systematics

- In GR, the cluster potential well Φ is shaped by the whole mass-energy content of the clusters: density and pressure separately

$$ds^2 = -e^{2\Phi(r)/c^2}\, c^2 dt^2 + \left(1 - \frac{2Gm(r)}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Metric of space-time inside a static, spherically symmetric system

- Galaxies are non relativistic, their velocity distribution depends only on Φ(r)
- Light trajectories respond to both Φ(r) and a relativistic term depending on m(r)



# Constraining the DM Equation of State

(Sartoris et al. 2014)

- EoS *parameter:* $$w(r) = \frac{p_r(r) + 2\,p_t(r)}{3\,c^2\rho(r)}$$

- $p_r(r)$, $p_t(r)$: radial and tangential pressure profiles fnct of m(r), Φ(r) and their derivatives
- ρ(r) is the density profile which depends on m(r): $\rho(r) = (1/4\pi)\, m'(r)/r^2$
- m(r), Φ(r) can be determined from independent determinations of $m_{kin}(r)$ and $m_{lens}(r)$

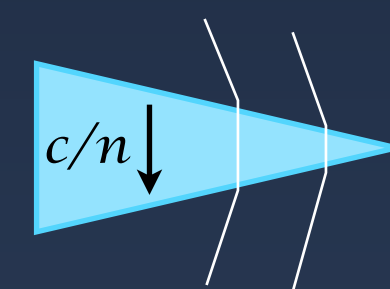
$$\nabla^2\Phi = \frac{4\pi G}{c^2}\left(c^2\rho + p_r + 2p_t\right)$$

$$m_k(r) = \frac{r^2}{G}\nabla\Phi(r).$$

in weak field approx

$$(2\Phi \ll c^2 \text{ and } 2mG/r \ll c^2)$$

Effective refraction index for lensing $$n = 1 - \frac{2\Phi_l}{c^2}$$ with: $$2\Phi_l(r) = \Phi(r) + G\int \frac{m(r)}{r^2}dr.$$




$$\rightarrow \quad m_l(r) = \frac{r^2}{G}\Phi_l'(r) = \frac{r^2}{2G}\Phi'(r) + \frac{m(r)}{2} = \frac{m_k(r)}{2} + \frac{m(r)}{2}$$

# Constraining the DM Equation of State

(Sartoris et al. 2014)

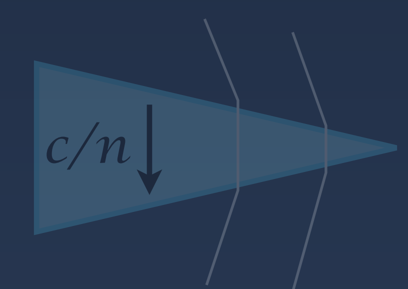
- EoS *parameter:*

$$w(r) = \frac{p_r(r) + 2\,p_t(r)}{3\,c^2 \rho(r)}$$

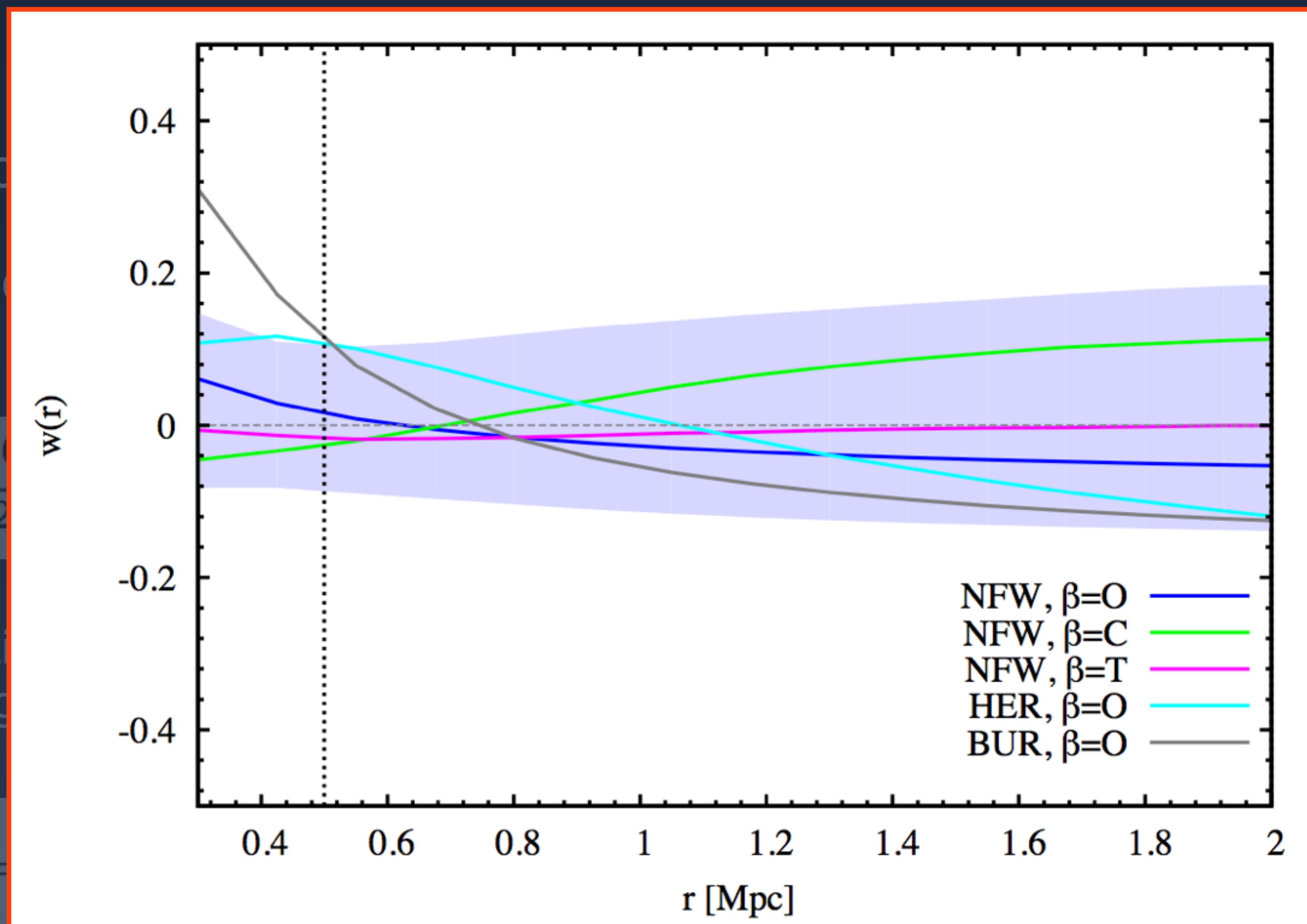
- $p_r(r)$, p
- ρ(r) is the c
- m(r), Φ(r) … s of m

$$\nabla^2\Phi = \frac{4\pi}{c^2} \ldots$$ in weak field approx … $\ll c^2$ and $2mG/r \ll c^2)$

Effective refracti… index for lensing




→ $m_l(r) = \ldots$



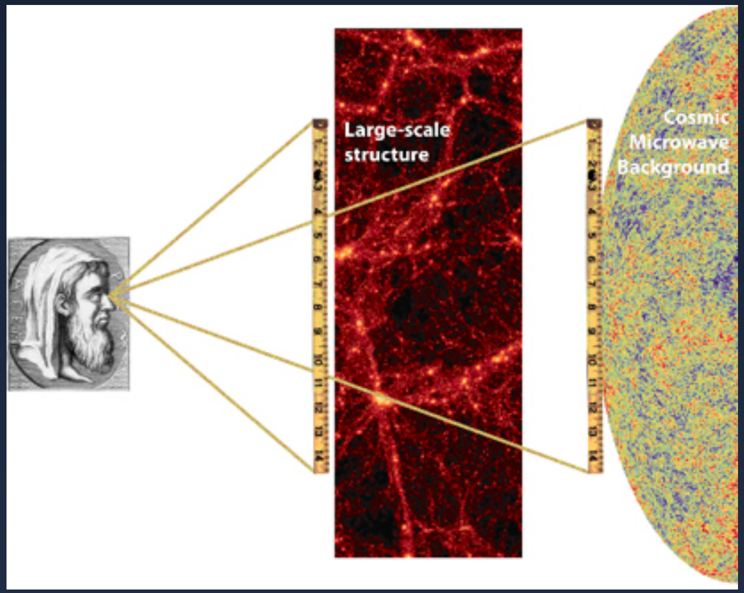

➔ For the cluster fluid, essentially DM (averaging over 0.5 Mpc−$R_{vir} \simeq$ 2 Mpc), we find:

$$w_{\rm DM} = 0.00 \pm 0.15({\rm stat}) \pm 0.08({\rm syst})$$

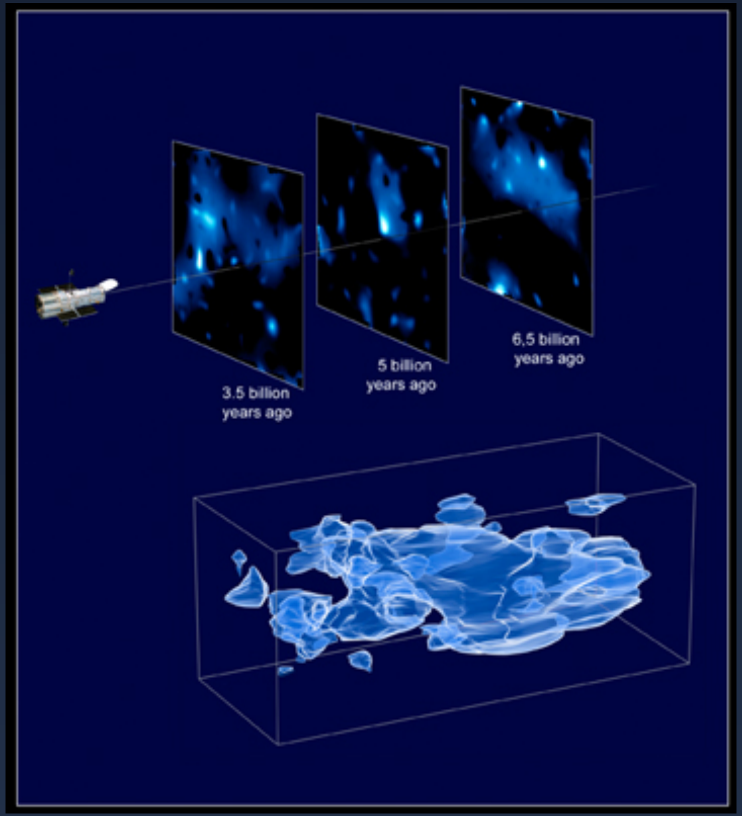
- Systematics will be better understood (and reduced?) when extended to 12 CLASH-VLT clusters

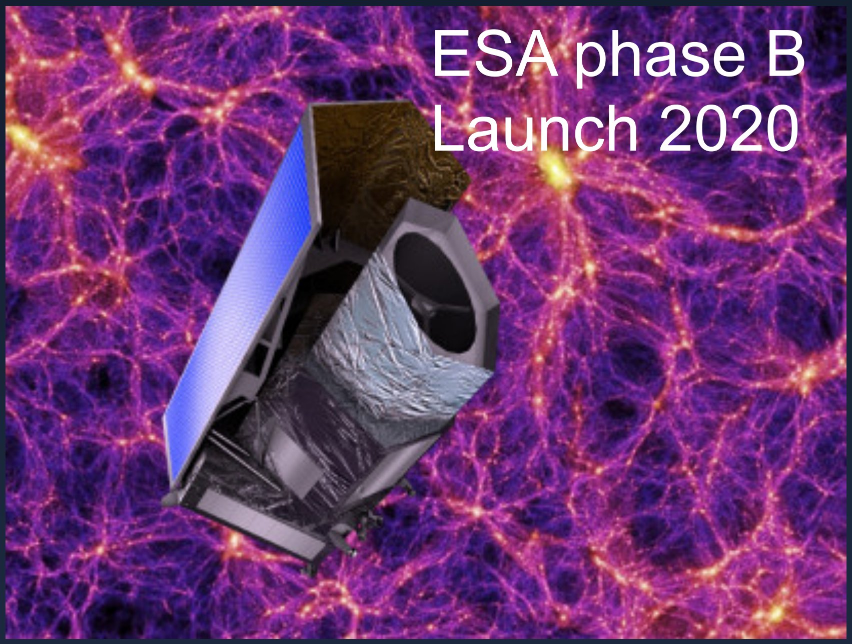
# EUCLID

Two primary cosmological probes:
BAO ($D_A$ and H at z=1−2) +




WL tomography
(matter power spectrum)



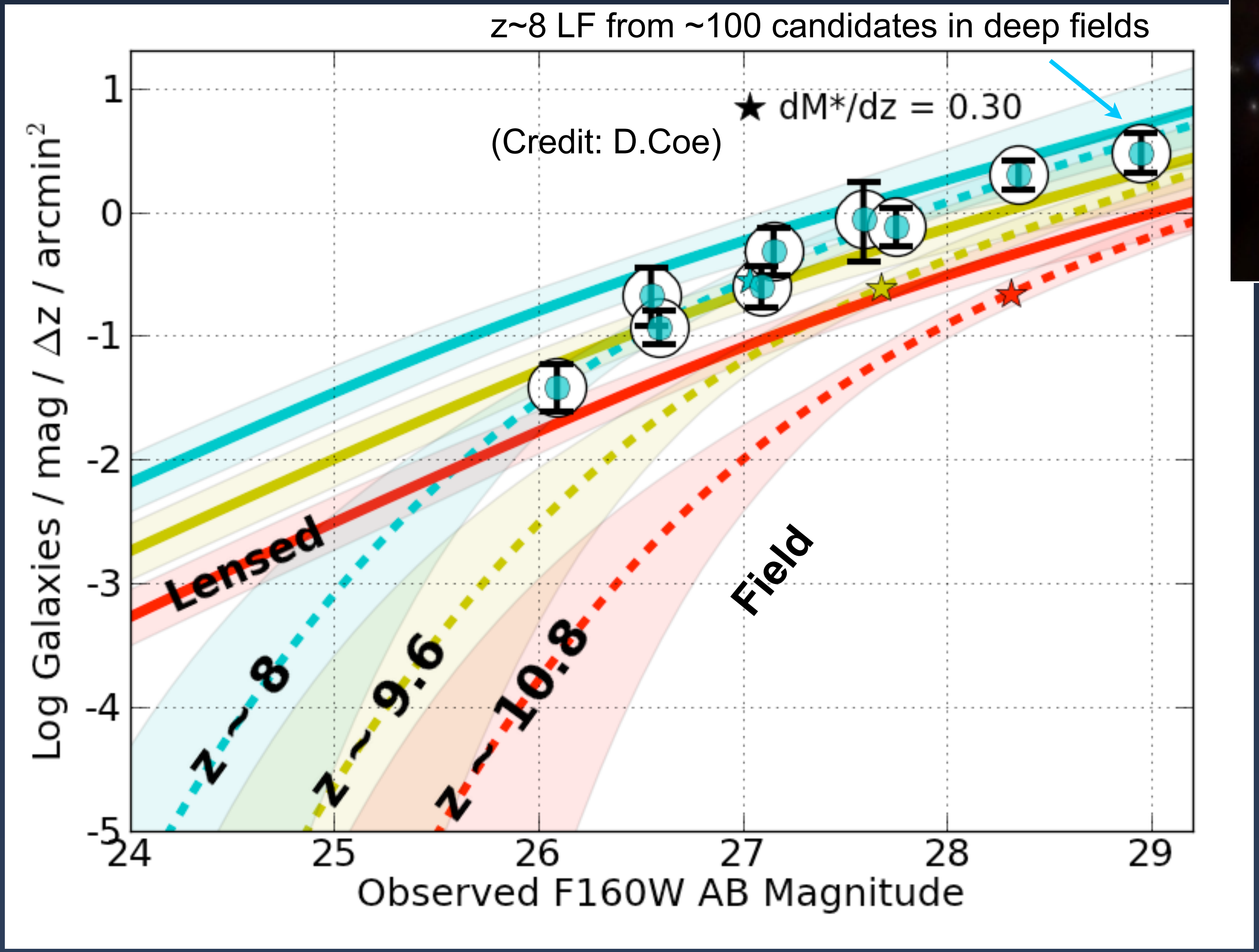



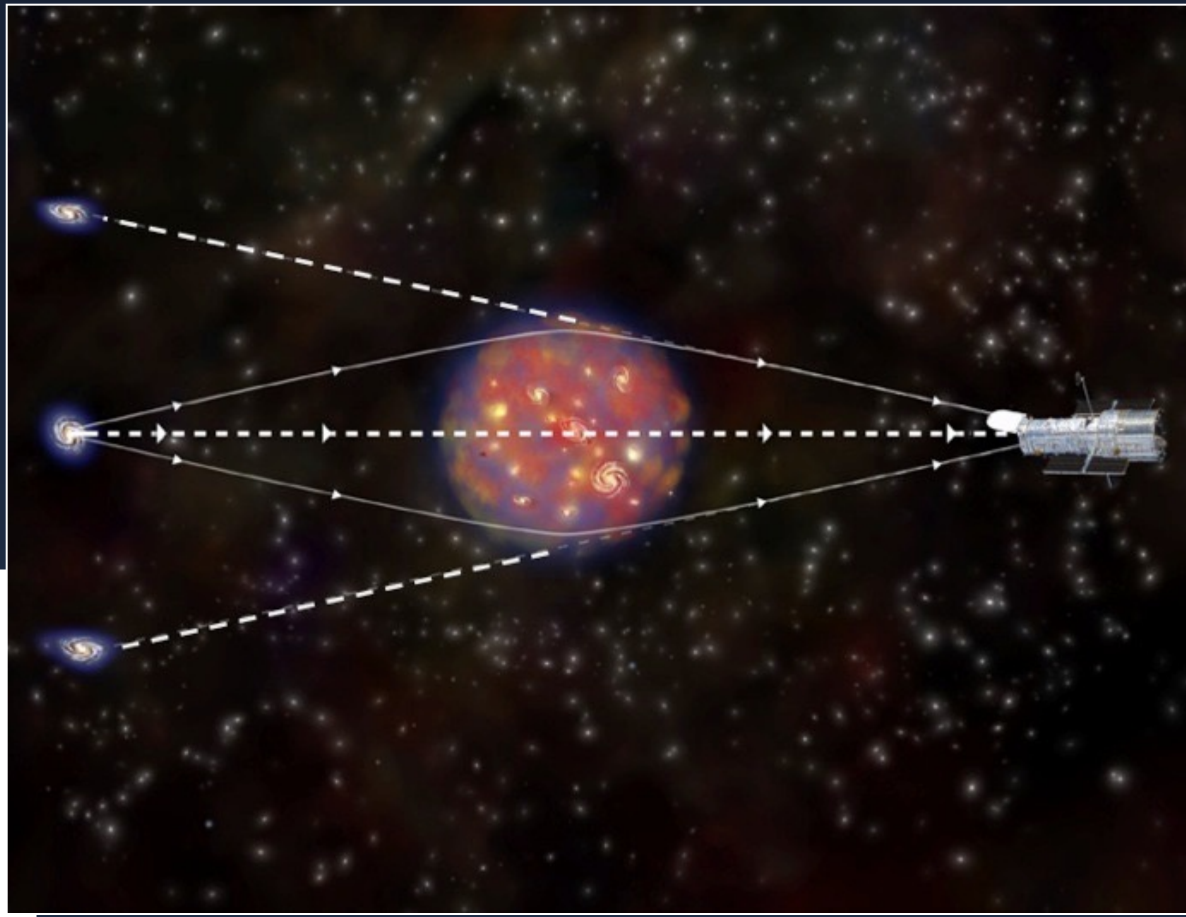

Cosmic shear: clumpiness of DM on different scales can be quantified statistically with correlation of shear signal along the l.o.s
→ measure projected matter PS
→ with photozs one can do tomography

- Imaging survey (opt + NIR) + slitless spectroscopic (NIR) survey (15,000 deg^2)
- Exploits geometry (BAO) and growth (WL + RSD + clusters) as cosmological probes
  - 50 millions of spectra (mostly Hα em. lines at z~1−2)
  - WL (cosmic shear) from optical channel (need photo-zs for lensing tomography)
  - $>10^5$ clusters (but mass calibration TBD)
- Goals:
  - $w_p$ at 1 %, $w_a$ at 5% [varying $w = w_p (a_p - a) w_a$ ]
  - distinguish modified gravity from dark energy (geometry and structure growth)
  - + Gaussianity of initial perturbation field, neutrino masses ($\sum m_\nu$)
  - + vast legacy science

# Galaxy Clusters as Cosmic Telescopes

- *Phenomenal progress over last 10 years driven by HST (ACS...WFC3/IR)*
- *Magnification (μ~3-100) significantly increases discovery efficiency for galaxies at fainter mags or/ and higher redshifts, but also the volume shrinks by $A_S \sim 1/\mu$*

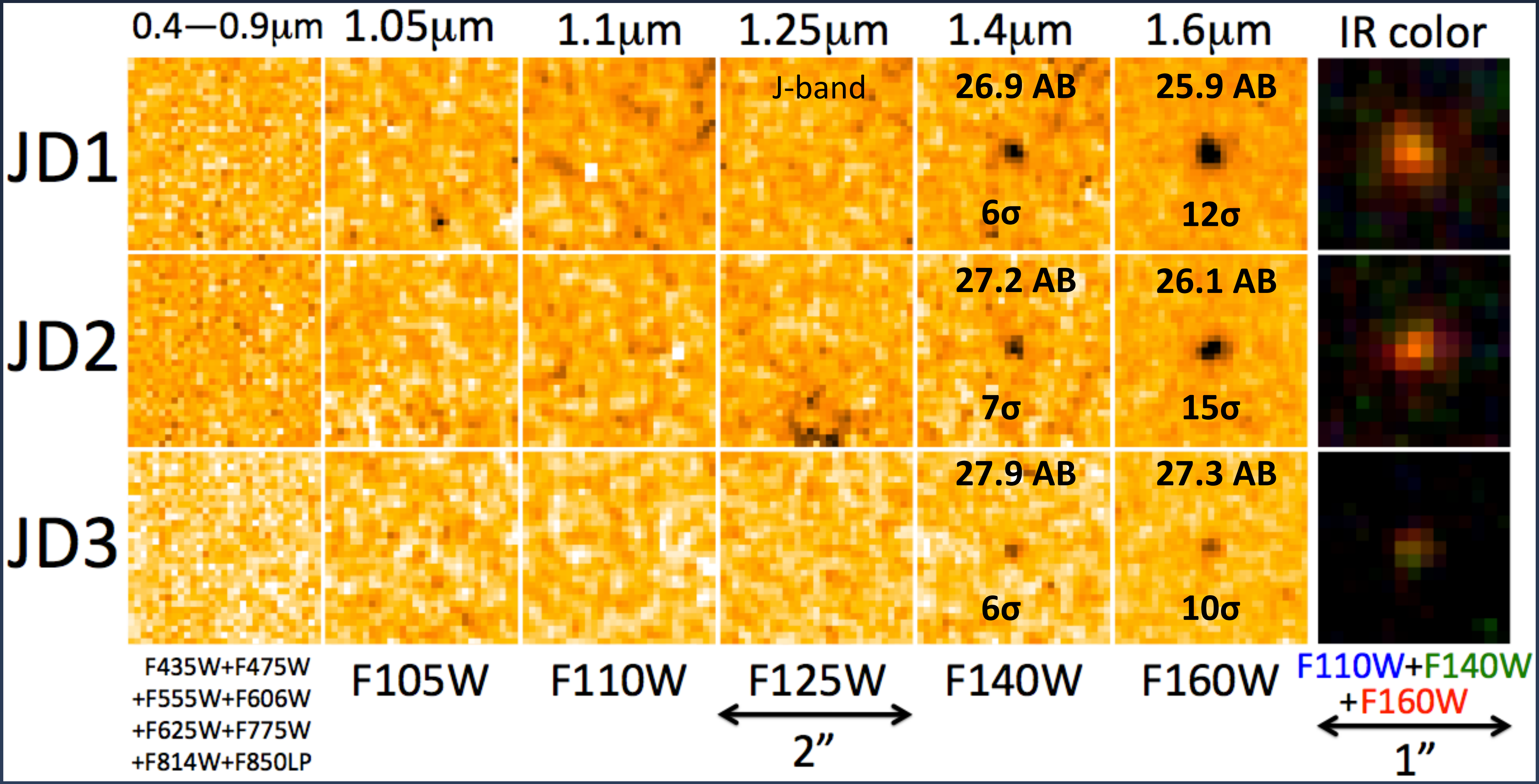


- *Lensing amplification gives access to the sub-L* galaxy population at z>6, in a complementary fashion to field studies (sensitive to L>~L*)*

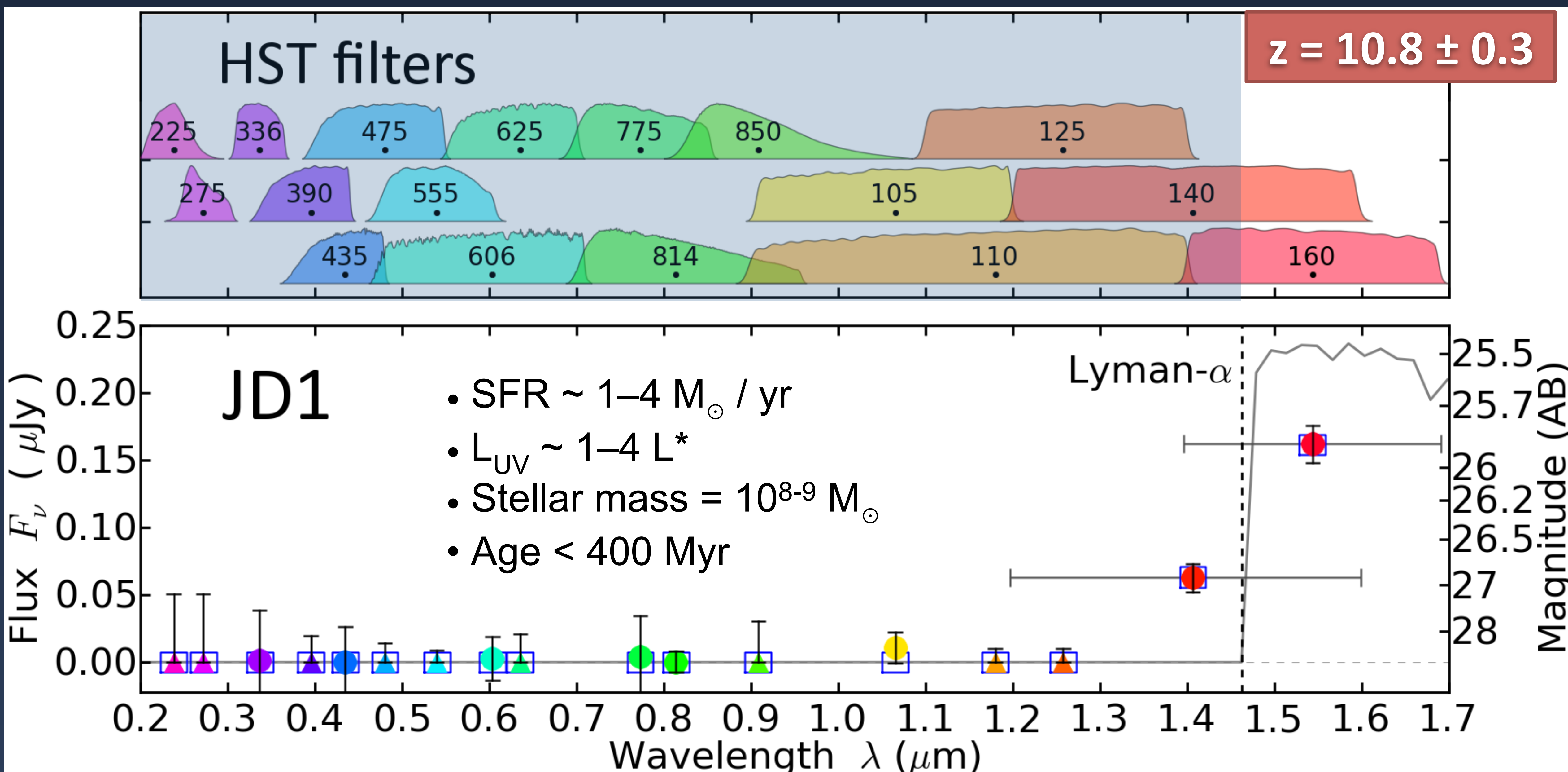
## MACS0407-JD (Coe et al. 2013)

- Each lensed images (with μ≈8, 7, 2) is observed only in the two reddest WFC3 filters + upper limits with IRAC 3.6μ and 4.5μ (JD1 ~3 mag brighter than HDF12 z~9 candidates)



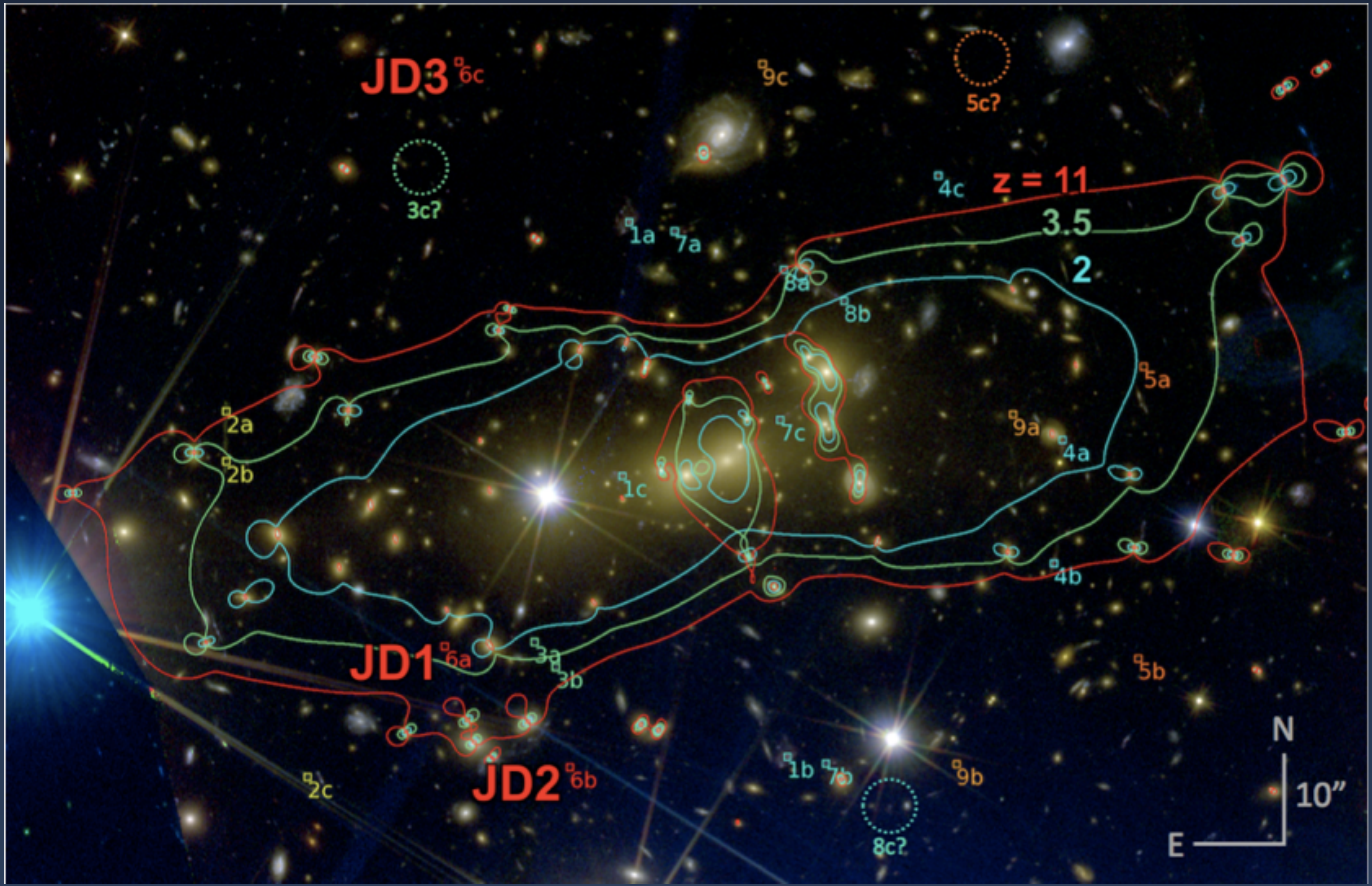
## MACS0407-JD (Coe et al. 2013)

- Each lensed images (with μ≈8, 7, 2) is observed only in the two reddest WFC3 filters + upper limits with IRAC 3.6μ and 4.5μ (JD1 ~3 mag brighter than HDF12 z~9 candidates)
- HST photometry is best fit by a starburst galaxy spectrum at z ~ 11, "all" other solutions extremely unlikely (z<9.5 interlopers ruled out at 7.2σ)



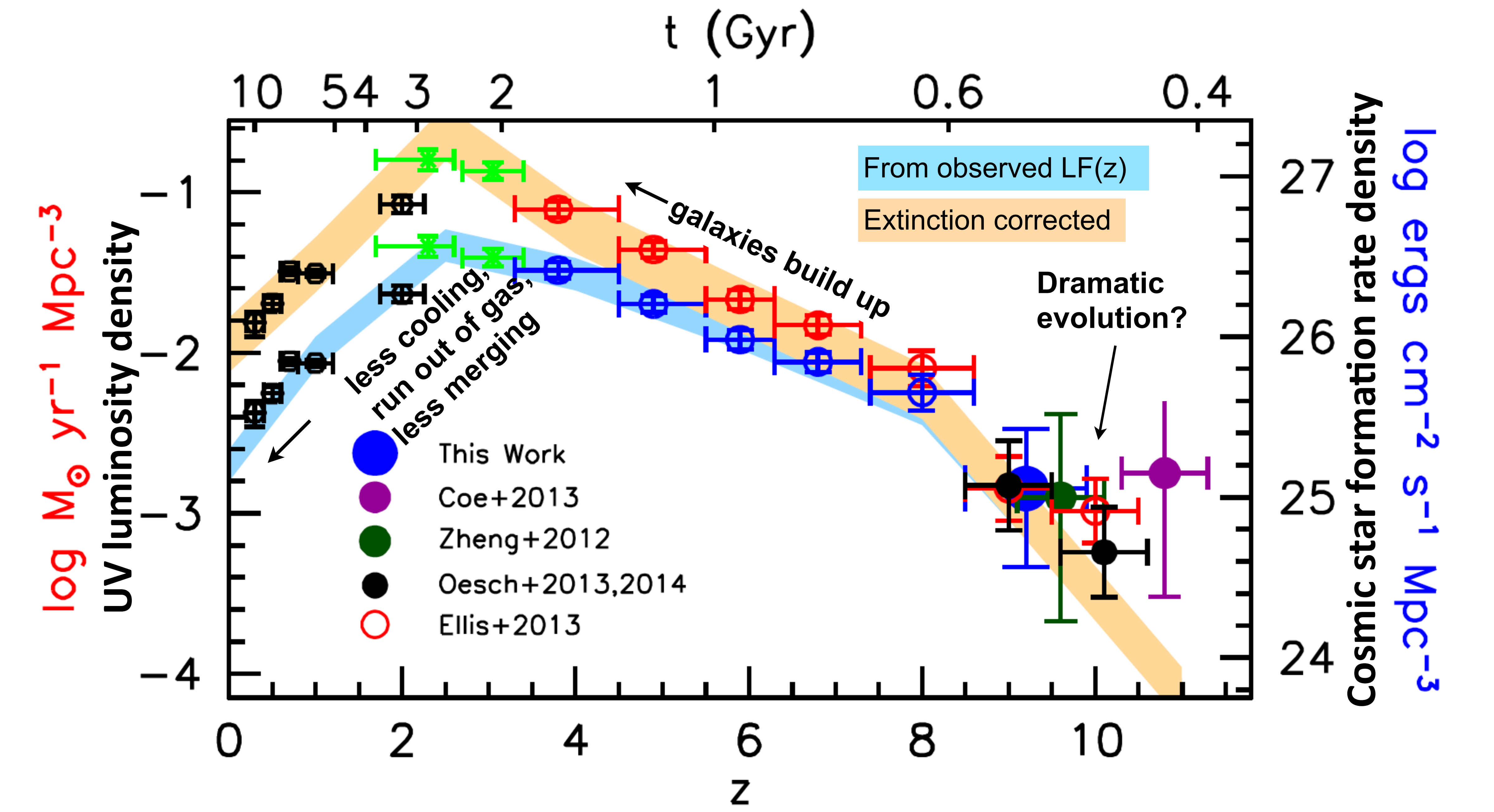
## MACS0407-JD (Coe et al. 2013)

- Each lensed images (with μ≈8, 7, 2) is observed only in the two reddest WFC3 filters + upper limits with IRAC 3.6μ and 4.5μ (JD1 ~3 mag brighter than HDF12 z~9 candidates)
- HST photometry is best fit by a starburst galaxy spectrum at z ~ 11, "all" other solutions extremely unlikely (z<9.5 interlopers ruled out at 7.2σ)
- Observed positions and fluxes are consistent with the lens models, based on 20 strongly lensed images of 8 other galaxies



**CLASH+Hubble Deep fields provide**

- the first census of galaxies ~500 Myr after the big bang
- first constraints on galaxy evolution at z > 8
- ...but more observations are required to confirm/rule out a rapid growth with important implications for reionization

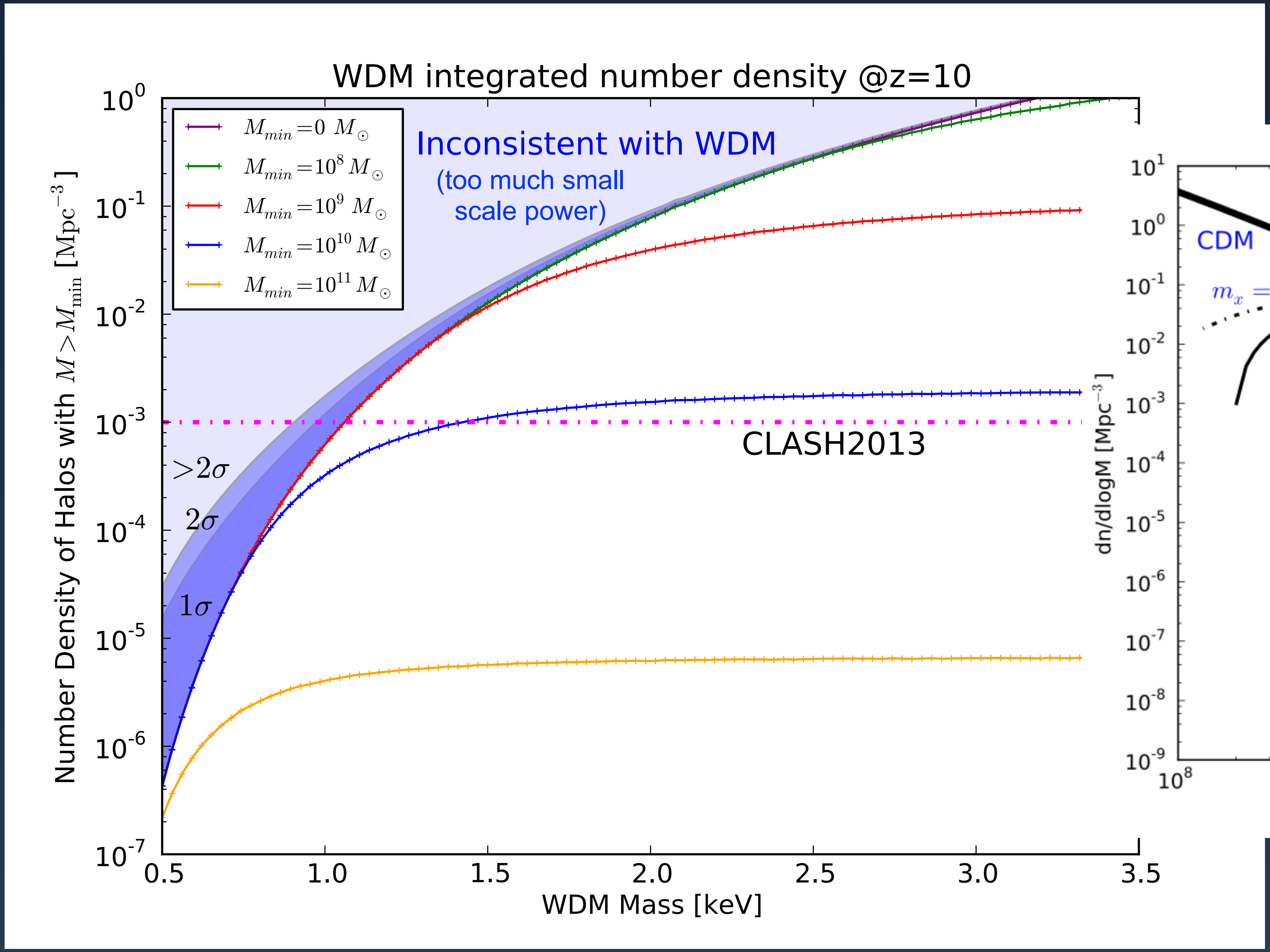


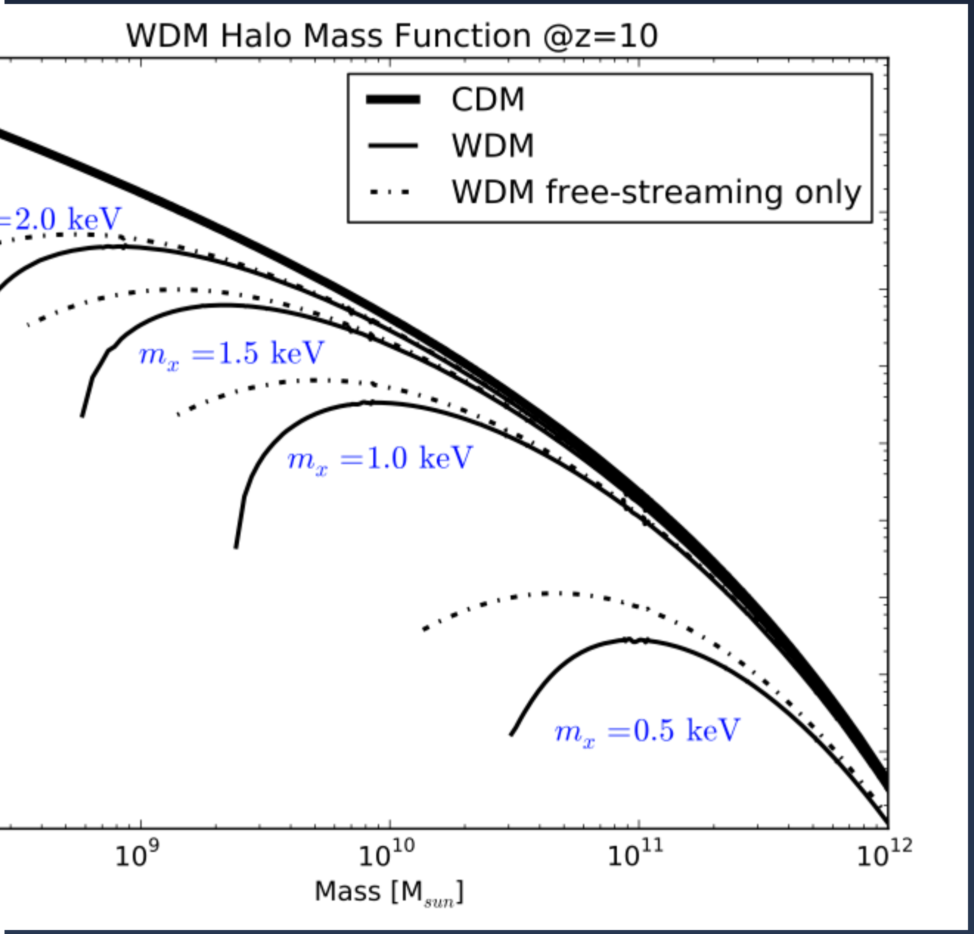

(Schiminovich+2005, Reddy&Steidel 2009, Oesch+ 2010, Bouwens+ 2007,11,12, Coe+2013)

# Independent constraints on the nature of DM from the number density of primordial galaxies

- Existence of galaxies at very high z implies significant primordial power on small scales (lower limit to the number density of collapsed DM halos)

Pacucci et al. (2013)



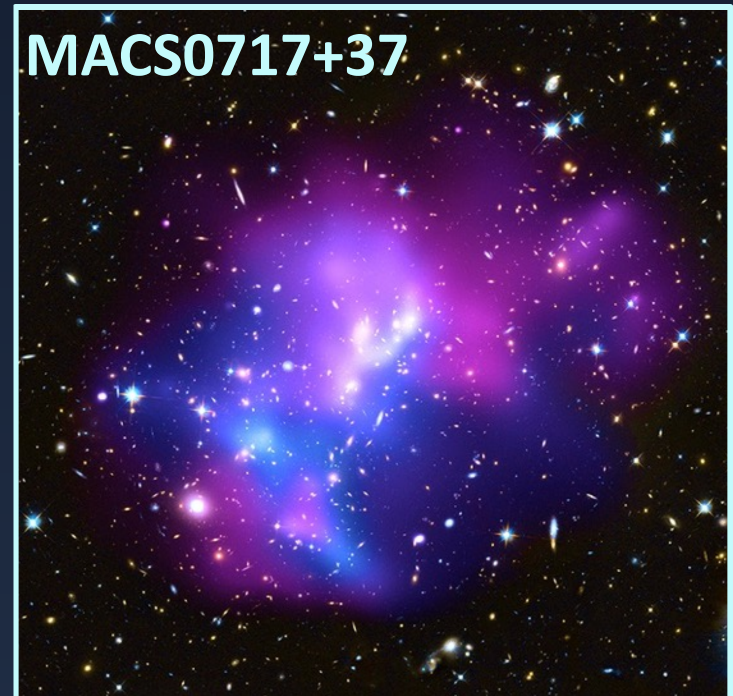




- Even only two galaxies at z~10 allow one to exclude WDM particles with $m_X$<1 keV
- Limit depends only on WDM halo mass function, not much on astrophysical modeling

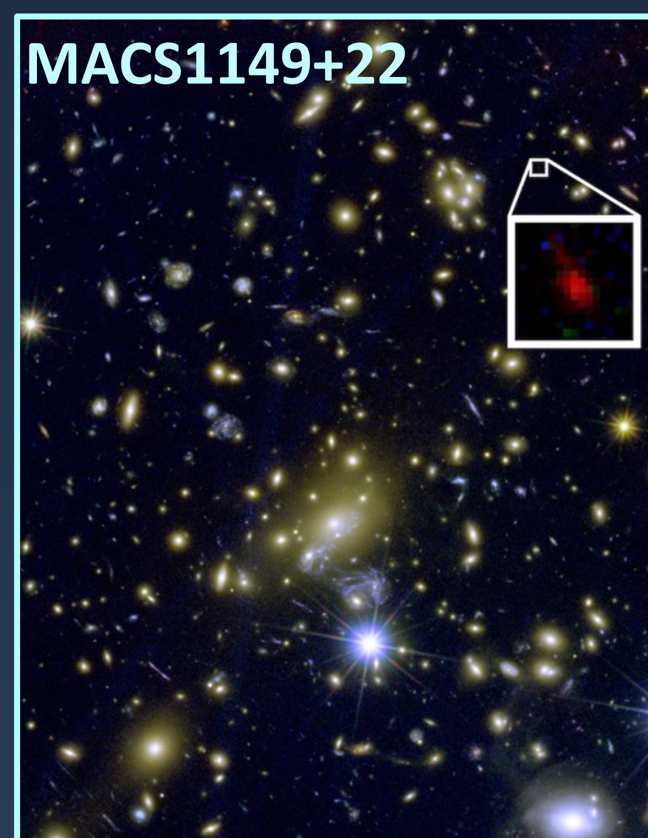
# Nature of DM from astro-particles studies

- DM is not baryonic (from cluster mergers) but also indirectly from CMB
- DM is to large extent collisionless ($\sigma/m$ upper limits from cluster mergers)
- DM is pressure-less and "cold", possibly "warm" but not too "hot" (non-relativistic at decoupling)
- Observed power of small-scale structure suggests $M_X > \sim 2$ keV (via free streaming scale)
- Large DM halo profiles match ΛCDM simulations, however significant deviations remain in the core and inner structure of the halos Improving maps of large DM halos should tell us whether deviations are simply due to baryonic physics
- No evidence yet (direct and indirect detection) that DM are WIMPs in the 100-1000 GeV scale. WIMPs match to thermal relic density ($\Omega_M$): miracle or fluke? production at accelerators hailed as next big goal..
- Time to broaden our searches and ideas ?

# Next:  The Frontier Fields





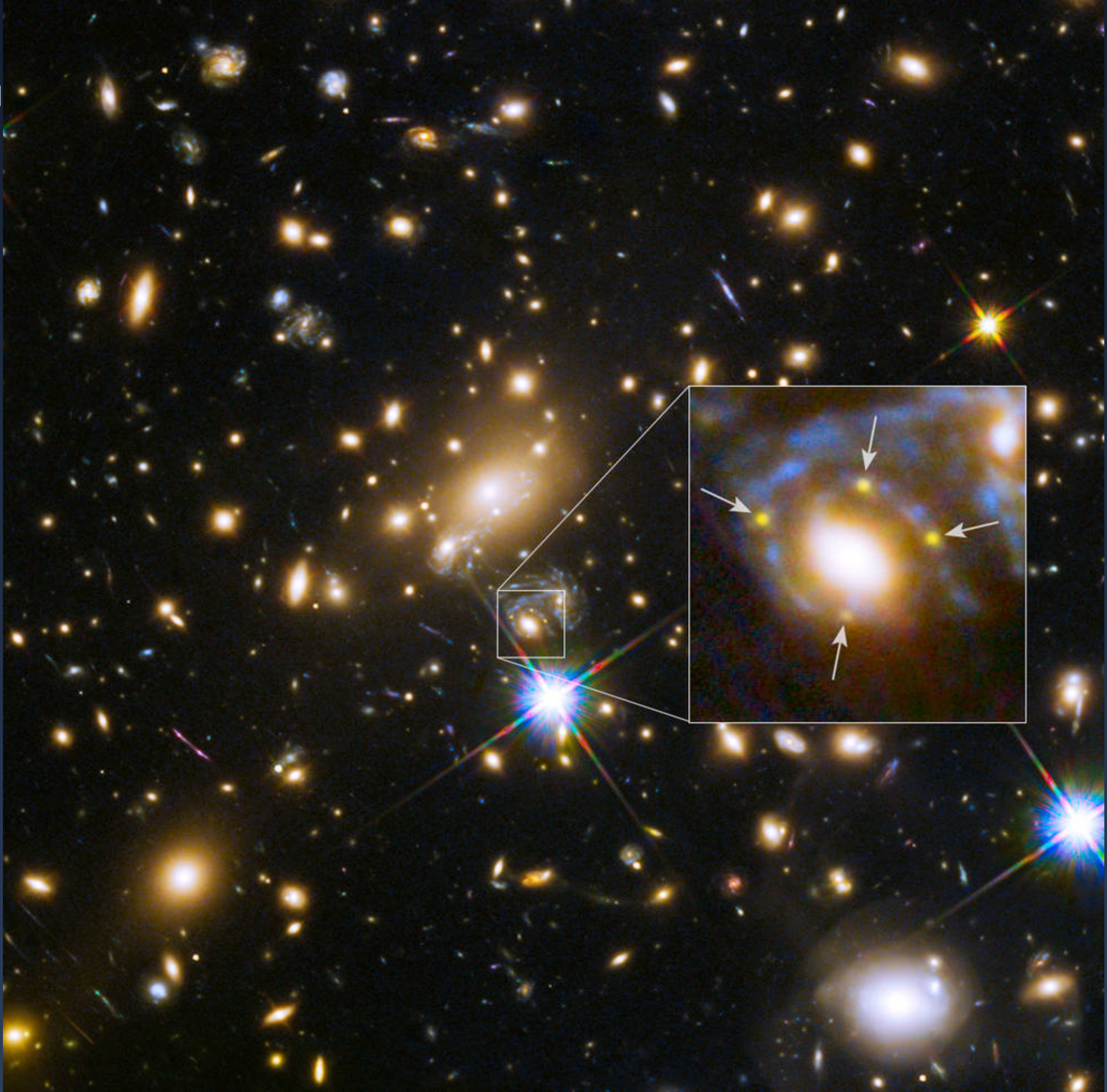
➔ 70 orbits ACS + 70 orbits WFC3/IR, 1.2 mag deeper than CLASH  (Fall 2013 – Fall 2016)

➔ Chandra large program for deep X-ray observations on-going

See Kelly et al. 2014
(astro-ph1411.6009)

and finally a multiply lensed SN…!

See Kelly
(astro-ph1



See Kelly
(astro-ph1